Non-Stationary Signal Reconstruction from Level-Crossing Samples using Akima Spline

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OUTLINE

- Introduction
- Signal sampling and algorithms
- Signal reconstruction
- Simulation and results
- Conclusions
INTRODUCTION

- The signal-driven data acquisition method for non-stationary signals, such as a level-crossing sampling (LCS) allows data sampling at a rate significantly below the Nyquist rate.

- It is achieved for the burst signals due to LCS properties as: no data sampling if a signal remains constant and sampling data only at the time instants when the signal is crossing the predetermined reference levels.

- Level crossing sampling method – a compromise between decreased amount of data by decreasing number of reference levels and precise signal reconstruction.

- The reduction of the signal data can be useful in certain signal processing applications including signal’s separation, biomedical signal processing and etc.
SIGNAL SOURCE

All original signals were taken from:

EEG time series Database https://epilepsy.uni-freiberg.de/freiburg-seizure-prediction-project/eeg-database.

Sampling frequency 256 Hz

Word length – 12 bits

24h signals, 6 channels
LIMITATIONS

- The signal length is 10 s and 5 min.
- Reference level's number is from 5 to 16.
- Reference levels are spaced equidistantly.
- Performance of two level-crossing sampling methods are evaluated.
- Performance of signal reconstruction using three types of splines are evaluated.
EXAMPLE OF THE NON-STATIONARY SIGNAL

![Graph of a non-stationary signal with time on the x-axis and amplitude on the y-axis. The signal shows peaks and valleys throughout the duration.]
LEVEL CROSSING (LC) APPROACH
LCS1 ALGORITHM

[Graph showing a time-amplitude plot with various peaks and troughs over time.]
LCS2 ALGORITHM
SIGNAL RECONSTRUCTION (INTERPOLATION)

- Piecewise polynomial curves – Akima, Hermite ("shape preserving"), Cubic spline

- The Akima interpolation is a continuously differentiable sub-spline interpolation. It is built from piecewise third order polynomials. Only data from the next neighbor points is used to determine the coefficients of the interpolation polynomial. There is no need to solve large equation systems and therefore this interpolation method is computationally very efficient.
SIGNAL RECONSTRUCTION (INTERPOLATION)

Piecewise cubic interpolating function is a cubic spline. The term “spline” refers to an instrument used in drafting. It is a thin, flexible wooden or plastic tool that is passed through given data points and defines a smooth curve in between. The physical spline minimizes potential energy subject to the interpolation constraints. The corresponding mathematical spline must have a continuous second derivative and satisfy the same interpolation constraints.
SIMULATION AND RESULTS

- Sampling is not uniform. Signal reconstruction is more complicated.
- Signal LCS is simulated using two MatLab programs LCS1 and LCS2.
- Then signals are reconstructed from LC samples using all three available interpolation routines for the same set of the signal samples.
- Performance is evaluated using evaluations metrics.
EVALUATION METRICS for RECONSTRUCTION QUALITY

1) Cross correlation

\[ r_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} \]

3) Compression ratio

\[ CR = \frac{S_{\text{orig}}}{S_{\text{lcs}}} \]

2) Root mean square error

\[ RMS = \sqrt{\frac{\sum_{i=1}^{n} (x_i - y_i)^2}{n}} \]

4) Percentage root mean square difference

\[ PRD = 100 \times \sqrt{\frac{\sum_{i=1}^{n} (x_i - y_i)^2}{\sum_{i=1}^{n} (x_i)^2}} \]
Evaluation of signal reconstruction methods

- Evaluation of the method's performance at first
- Cross correlation metric by correlating values of original and reconstructed signal for different number of reference levels

![Graph showing cross correlation coefficient for different number of levels for Akima, Pchip, and Cubic methods.](image-url)
Cross correlation of EEG 10 s reconstructed signal versus number of reference levels for Akima and Hermite splines, and LCS algorithms
Cross correlation of EEG 5 min long reconstructed signal

![Graph showing cross correlation coefficient for different methods (Akima1, Akima2, Pchip1, Pchip2) across varying number of levels. The graph plots the cross correlation coefficient on the y-axis against the number of levels on the x-axis. The methods show different trends and values as the number of levels increases.]
Root mean square error for EEG reconstructed signal versus number of reference levels for Akima and Hermite splines
PRD for EEG reconstructed signal versus number of reference levels for Akima and Hermite splines
Compression ratio versus number of reference levels for LCS1 and LCS2 algorithms

Number of levels vs. Compression ratio for LCS1 and LCS2 algorithms.
Percentage root mean square difference for EEG reconstructed signal samples versus number of reference levels for LCS1 and LCS2 algorithms
CONCLUSIONS

- Akima spline has proved to be the most effective for non-stationary signal reconstruction from level-crossing samples.

- Hermite spline is demonstrating good performance for all metrics but is not exceeding Akima spline.

- The quality of reconstructed signals is depending from the used number of reference levels.

- Any increase of the number is causing improvement in all metrics. The exception is the compression ratio. Therefore the equilibrium between method’s performance and method’s compression ratio should be observed.
QUESTIONS?

Thank you for your attention!

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