Reconstruction of sequences of arbitrary-shaped pulses from its low-pass or band-pass approximations using spectrum extrapolation

Modris Greitans and Rolands Shavelis
Institute of Electronics and Computer Science
14 Dzerbenes Str., Riga LV-1006, Latvia

Summary

The paper discusses the problem of processing short-time pulse sequences. Since the spectrum of such signals occupies wide range, it is difficult to sample them at the Nyquist rate. Instead, the approach used for processing signals with finite rate of innovation is employed – sequence of pulses is filtered with low-pass or band-pass filter before sampling. A waveform reconstruction method based on spectrum extrapolation in an iterative way is proposed. An application of the results can be used in ultra wideband impulse radio systems.

Introduction

Information in ultra wideband impulse radio (UWB-IR) systems is transmitted by generating extremely short pulses with durations less than 1 ns and thus occupying large frequency bandwidth.

Digital data to the analog pulses is added by means of modulation – pulse position and pulse shape modulations. There can be different situations at the receiver - all pulses are with the same shape or multiple pulse types are used.

As UWB pulses are extremely short, it is difficult to provide the sampling rate determined by the bandwidth of transmitted signal to decode the received signal. In [1,2] it is shown that it is possible to recover a non-bandlimited signal from uniform samples of its low-pass approximation, if the signal has a finite rate of innovation – number of parameters per unit time required to model the signal. The reconstruction is based on the use of an annihilating filter, and the samples have to be taken at rate above the rate of innovation.

The annihilating filter method is suitable for recovery of stream of Diracs as well as for pulses of one type (Fig.1). If the pulses have different shapes and durations (Fig.2a), an alternative method based on spectrum extrapolation using signal dependent transformation kernel [3] is proposed.

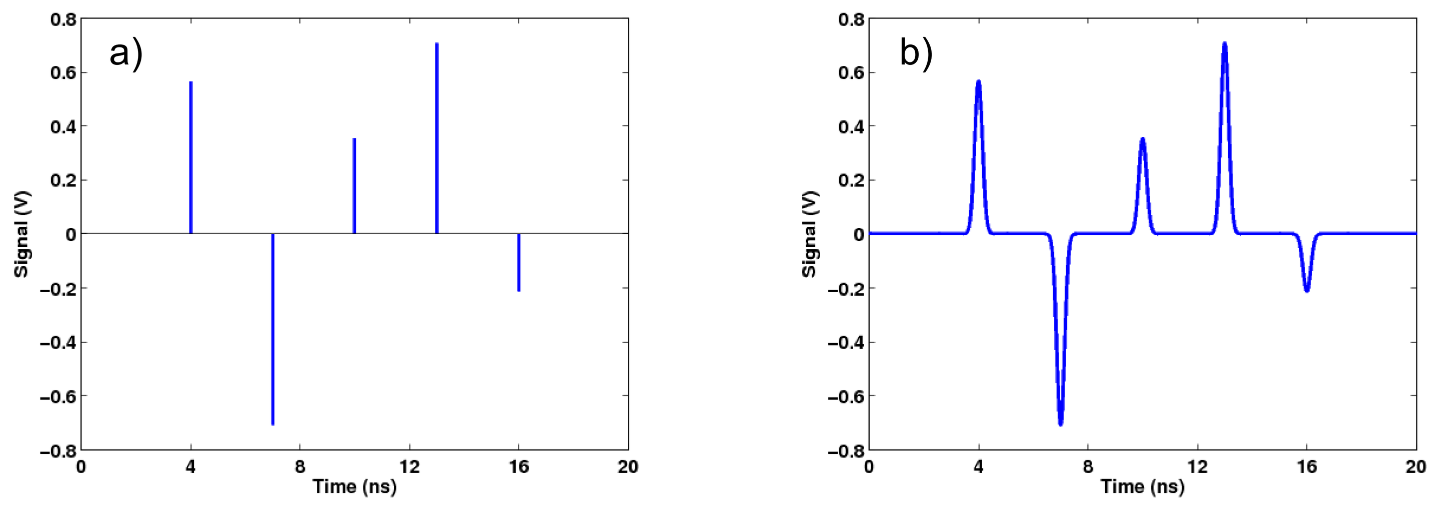

Figure 1. Examples of signals with finite rate of innovation: a) stream of Diracs, b) stream of Gaussian pulses.

Spectrum extrapolation approach

The method used for processing band unlimited signals with finite rate of innovation employs bandwidth restriction of filtering before sampling. The bandwidth of the original signal is thus decreased and the samples are taken at lower sampling rate. To reconstruct the signal it is necessary to recover the whole bandwidth in the frequency domain.

Let us have an UWB signal $x(t)$ consisting of stream of different pulses. The length of the shortest pulse is τ_p. Now, if the signal is sampled at rate $5/\tau_p$ providing N samples $\mathbf{x}=[x(0),x(1),\dots,x(N-1)]$ then each pulse is represented by at least 5 samples. The discrete Fourier transform (DFT) of $\mathbf{x}$ allows to obtain spectrum coefficients

$$\mathbf{X}(1\times N)\ :\ X(k)=\mathbf{x}\,\mathbf{W}^k$$

where $\mathbf{W}^k=[e^{-j2\pi k0/N},e^{-j2\pi k1/N},\dots,e^{-j2\pi k(N-1)/N}]^T$, and $k=0,\dots,N-1$. To find the original signal samples, the inverse DFT of $\mathbf{X}$ is taken.

After ideal filtering, only $M<N$ spectrum coefficients $\mathbf{X}_M=[X(r),X(r+1),\dots,X(r+M-1)]$ of the signal remain, while the rest $N-M$ coefficients become zero. In lowpass filtering case $r=0$. Recovering of the whole set of the spectrum coefficients from $\mathbf{X}_M$ is based on Capon or minimum variance (MV) filter approach, which requires the knowledge of spectrum autocorrelation matrix [4]. In general, when signal consists of stream of different pulses, the spectrum autocorrelation is not known in advance and thus is estimated in an iterative way [5,6]. The algorithm is:

$$\mathbf{R}_i(M\times M)\ :\ R(m,l)=\hat{\mathbf{P}}_{i-1}\mathbf{W}^{l-m}, \tag{1}$$

$$\hat{\mathbf{x}}_i(1\times N)\ :\ \hat{x}_i(n)=\frac{\mathbf{X}_M\mathbf{R}_i^{-1}\mathbf{W}_M^{-n}}{(\mathbf{W}_M^n)^T\mathbf{R}_i^{-1}\mathbf{W}_M^{-n}}, \tag{2}$$

$$\hat{\mathbf{P}}_i(1\times N)\ :\ \hat{P}_i(n)=|\hat{x}_i(n)|^2 \tag{3}$$

The elements of autocorrelation matrix $\mathbf{R}_i$ are calculated from the signal power $\hat{\mathbf{P}}_{i-1}$, $\hat{\mathbf{x}}_i$ is the recovered signal after iteration i, $\mathbf{W}_M^n=[e^{-j2\pi rn/N},e^{-j2\pi(r+1)n/N},\dots,e^{-j2\pi(r+M-1)n/N}]^T$, and $i=1,2,3,\dots$ are iteration numbers. The estimator $\hat{x}_i(n)$ is considered as found and the iteration as completed when the power $\hat{P}_i(n)$ does not alter from iteration to iteration or the changes are small by comparison with a selected criterion. The initial conditions in the absence of a priori information are determined from the inverse DFT signal estimator

$$\hat{P}_0(n)=\left|\frac{1}{M}\mathbf{X}_M\mathbf{W}_M^{-n}\right|^2$$

Extrapolation $\hat{\mathbf{X}}_i$ of $\mathbf{X}_M$ is provided by taking DFT of $\hat{\mathbf{x}}_i$.

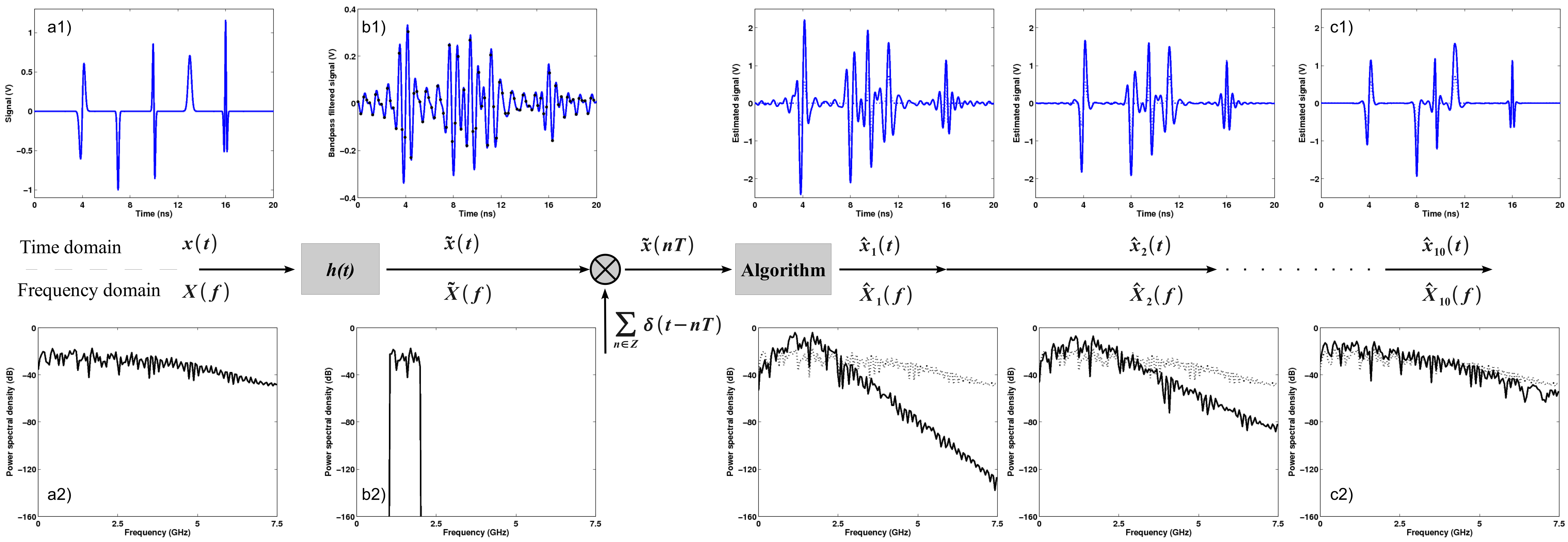

Figure 2. Processing of stream of pulses: a1) input signal of multiple pulse types, a2) spectrum of the input signal, b1) bandpass filtered version of the input signal, b2) spectrum of the bandpass filtered signal, c1) reconstructed signal (solid line), c2) spectrum of the reconstructed signal (solid line).

Simulation results

Signal consisting of 2 Gaussian pulses, 2 Gaussian monocycles and 1 Gaussian doublet is considered (Fig. 2a1). The time-scaling factors for the second, third and fifth pulses are 0.1 ns, while for the first and fourth pulses it is 0.2 ns. The amplitudes are chosen to ensure the energy of all pulses is equal.

The signal is bandpass filtered removing frequencies lower than 1 GHz and higher than 2 GHz. The filtered signal (Fig. 2b1) is sampled to obtain processing data. Given 81 sampling values, $M=21$ spectrum coefficients $\mathbf{X}_M$ from 1 to 2 GHz are found. By putting $\mathbf{X}_M$ in algorithm (1), (2) and (3), $N=301$ values of the recovered signal are calculated and 151 spectrum coefficients up to frequency 7.5 GHz are estimated. The results after 10 iterations are shown in Fig. 2c1 and c2. Time locations of reconstructed pulses exactly correspond to the original signal, while the amplitudes and lengths are reconstructed with some distortions.

Conclusions

The method for recovering UWB-IR signals consisting of arbitrary-shaped pulses is proposed. The reconstruction uses uniform samples of lowpass or bandpass filtered signal to obtain spectrum coefficients, which are further extrapolated using iterative update of spectrum autocorrelation function. Passband of the filter should correspond to the band with the highest signal power.

As UWB is low-power communication it can be efficiently used in wireless sensor networks where data is obtained by event driven sampling techniques such as, for example, level-crossing sampling. In this case, instants of the events occurrences can be coded by the pulse position on the time axis, while type of the events (for example, upward or downward crossing) can be coded by shape of the pulse.

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For further information

Please contact modris_greitans@edi.lv or shavelis@edi.lv

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