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Precise Measurement of Event Flow Time Coordinates Based on the Digital Processing of a Triggered Relaxation Oscillator Wave Train

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Abstract—This article is dedicated to the study of the interpolative measurement of event occurrences based on the digital processing of a relaxation oscillator generated during the event occurrence. It is shown that the use of linear algorithms for estimating the performance of a regression oscillation wave train model allows one to guarantee picosecond accuracy of the interpolative measurements. The technique's accuracy capabilities are illustrated by the results of actual measurements and computer modeling.

Keywords: event timing, interpolator, oscillation wave train, triggered relaxation oscillator, regression model, least-squares method, time interval, nonlinearity

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1. INTRODUCTION

The time coordinates of an event flow are event occurrence *time instants*. The way these are measured is referred to as *event timing*, and these measurements are made using discrete (rough) and interpolative (correction) techniques. The former type of measurement is confined to registering the actual condition of the meter of the increment interval (reference frequency) periods. The latter type of measurement is intended for refining the position of an event within the interval of the rough measurement discrete and is applied with the help of an interpolator realized as a hardware and software subsystem in a precision event timer.

Interpolation based on digital signal processing is very promising in terms of the achievable accuracy and resolving power [1, 2]. These techniques have the following basic steps: the generation of a specific analog signal during the event occurrence, the discretization and digitalization of its instant signal values, and the further digital processing of the computations, which results in the estimation of the signal *time position* within a discrete interval T_s of the rough measurement.

Works [3, 4] consider interpolative measurement using algorithms for estimating the location of the center of a unipolar analog signal according to its samples. It is shown that the estimations of the *time position* of the sample sequence center of an analog signal are nonlinearily dependent on the event occurrence instants. To compensate the nonlinearity of the interpolative measurement, additional correction of the time position estimations is therefore necessary. A specific corrective function for estimating the signal center time position helps determine the interpolative estimation of the event occurrence instants. The function may be derived after the obligatory and very complex calibration of the event timer and upon identifying the transfer function of the interpolator.

This work considers the results of the theoretical and experimental research of interpolative measurement that does not require extra correction of the estimations of the analog signal time position and therefore makes it unnecessary to preliminarily identify the transfer function of the interpolator. The studied technique is based on estimating the parameters of the regression model of an analog signal according to a sequence of samples. The processing is finding a best fit of the regression function describing the *waveform* of an analog signal to signal samples. The regression function coefficients chosen as informative parameters of the modeled signal must depend on the level of the signal shift relative to a certain initial moment of time.

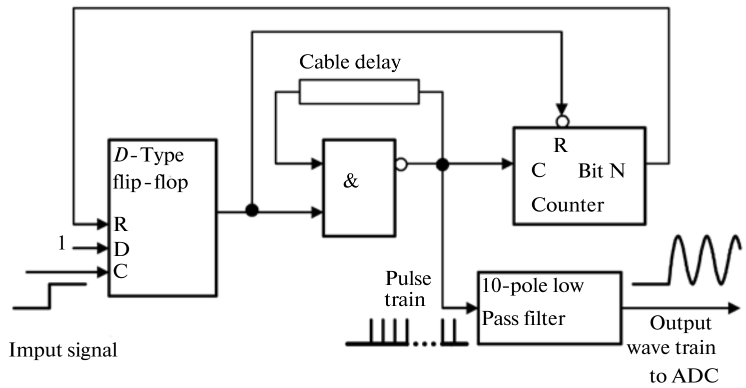

Fig. 1. Converter of input events into the output oscillation wave train of the triggered relaxation oscillator.

The estimation $\hat{\gamma}$ of the parameter vector γ of the regression function $g(t; \gamma)$ is determined by minimizing of some error functional

$$\hat{\gamma} = \arg\min_{\gamma} \Im(\gamma), \quad (1)$$

where

$$\Im(\gamma) = \sum_{k=1}^{N} [u_k - g(t_k; \gamma)]^2, \quad (2)$$

i.e., the estimation minimizes the sum of the squared signal sample $\{u_k = u(t_k)\}_{k=1}^{N}$ deviations measured during the instants $\{t_k = kT_s\}_{k=1}^{N}$ from the values of the regression function $g(t_k; \gamma)$, where T_s is the sampling frequency period amounting to the interval of the rough measurement discrete. Such values of the regression coefficients are selected to estimate the parameters of the modeled signal, which allow the model to harmonize as best as possible with the signal samples in terms of the mean square deviation (least-squares method). estimations of the regression coefficients are used to determine the time shift from the inception of a relevant interval of rough measurement discrete.

The algorithm of estimating the regression function coefficients with the help of the least-square method was considered earlier in article [5] and was exemplified by the processing of exponentially fading oscillations of an impact excitation loop. The use of computer modeling allowed revealing the possibility of picosecond accuracy without correcting the time shift values.

This article considers algorithms for processing analog signal samples. The signal referred to is a *wave train* generated by a triggered relaxation oscillator when each event of the flows occurs.

2. REGRESSION MODEL OF A TRIGGERED RELAXATION OSCILLATOR WAVE TRAIN

It is known that a wave train is a temporally or spatially limited sequence of any oscillations that are not necessarily harmonic. To generate an oscillation wave train during an event occurrence, it is necessary to use a *triggered relaxation oscillator*. A relaxation oscillator is an oscillation generator where the active part operates in the key “on/off” mode. It should be noted that the device in question is a nonlinear dynamic system invariant to shifting: the time shift of an input signal (event) for a certain value generates a corresponding shift of the output signal of the same amount. The specified feature ensures that the oscillations in a triggered relaxation oscillator are the same during each event occurrence.

A triggered relaxation oscillator was used as the basis for developing a converter of input events into output oscillation wave trains (see Fig. 1). The new device consists of a *D-Type Flip-Flop* logical element “AND” (*gate &*) with a *cable delay* in the delayed feedback, an pulse *counter*, and a *10-pole low pass filter.*

The converter operates as follows. The D-trigger switches on to the state of the logical “one” along the input signal front, i.e., the event occurrence mark, and the logical element “AND” and the pulse counter are therefore unblocked. The “AND” element and the cable delay line constitute a triggered relaxation

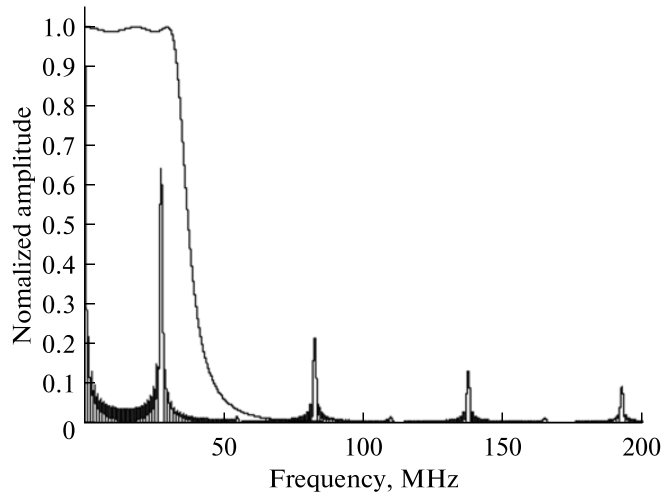

Fig. 2. Amplitude spectrum of a series of 25 rectangular pulses with a repetition frequency of 27.9 MHz at the input of the low pass filter (solid line) and the amplitude–frequency response of the low pass Chebyshev filter (dashed line).

oscillator that starts to produce rectangular pulses with a period equal to a doubled sum of the cable and logical element delays. After the set number of countings is reached, the counter generates a logical unit according to its output charge Bit N. The D-trigger and the counter are reset, and pulse generation stops. The series of pulses generated at the output of the relaxation oscillator goes through a low pass *Chebyshev Filter* with a cutoff frequency exceeding the pulse repetition in the series. A very smooth oscillation wave train is therefore generated at the output of the filter and fed to the input of a 12-digit analog-to-digital converter (ADC). The wave train sampling frequency is 100 MHz, which corresponds to a period of $T_s = 10$ ns.

When an event occurs, the output oscillation wave train is shifted by a certain value relative to the set frame of reference points, i.e., a position along the time axis of the sampling instants.

The shape of an oscillation wave train at the output of the low frequency filter is generally not an ideal sine wave. The high-frequency components of the spectrum of a series rectangular pulses cannot be fully suppressed by the real low frequency filter (see Fig. 2) and must be taken into account in a modeled oscillation wave train by the corresponding selection of a regression function.

The regression function approximating the oscillation wave train consists of a finite range of harmonic components of frequencies divisible by the main oscillation frequency and includes odd as well as even harmonics:

$$g(t;\gamma) = c_0 + \sum_{n=1}^{m}(a_n \cos n\omega_c t + b_n \sin n\omega_c t), \tag{3}$$

where c_0 is a constant component of the oscillation wave train, $\gamma = [c_0, a_1, b_1, \ldots, a_n, b_n, \ldots, a_m, b_m]^T$ is the vector of the estimated regression coefficients, ω_c is the basic frequency of the wave train oscillations (wave train fill frequency), and m is the number of frequency elements. In this study, the upper index "T" indicates hereafter the transposition of a vector or a matrix, while ω and Ω indicate angular frequencies. Introducing the time shift parameter τ, a regression function can be written as follows:

$$g(t-\tau;\gamma^{(0)}) = c_0 + \sum_{n=1}^{m}\left[a_n^{(0)} \cos n\omega_c(t-\tau) + b_n^{(0)} \sin n\omega_c(t-\tau)\right], \tag{4}$$

where the upper index, i.e., the zero in round brackets, indicates the "starting" values of the regression function coefficients at null shift; $\tau = 0$; and

$\gamma^{(0)} = \left[c_0, a_1^{(0)}, b_1^{(0)}, \ldots, a_n^{(0)}, b_n^{(0)}, \ldots, a_m^{(0)}, b_m^{(0)}\right]^T$ is the vector of the "starting" values of the regression function coefficients. The dependence of the regression coefficients a_n and b_n on the time shift τ is expressed as follows:

$$a_n = a_n^{(0)} \cos n\omega_c \tau - b_n^{(0)} \sin n\omega_c \tau \tag{5}$$

$$b_n = b_n^{(0)} \cos n\omega_c \tau + a_n^{(0)} \sin n\omega_c \tau. \tag{6}$$

The estimated informative parameters of the model (3) are the amplitudes $\{a_n, b_n\}_{n=1}^{m}$ of the quadrature harmonic components with the frequencies $n\omega_c$ and the wave train constant component c_0. The regression model (3) is linear in terms of all the estimated parameters. The parameters are estimated according to the minimum of functional:

$$\Im(\gamma) = \sum_{k=1}^{N} \left\{ u_k - \left[c_0 + \sum_{n=1}^{m} (a_n \cos n\omega_c kT_s + b_n \sin n\omega_c kT_s) \right] \right\}^2, \tag{7}$$

where $\{u_k\}_{k=1}^{N} = \{u(kT_s)\}_{k=1}^{N}$ is a sequence of wave train samples at the sampling instants $\{kT_s\}_{k=1}^{N}$. The parameters $\{a_n, b_n\}_{n=1}^{m}$ contain the data on the amplitude and the phase of the n frequency element of the wave train. A phase expressed in units of time is the start of the oscillation wave train shifted in time within the interval $[0, T_s)$. The model parameter ω_c (the main oscillation frequency) has no data on the wave train time shift and is therefore not informative. The value of the main wave train oscillation frequency is presumed to be known before the regression coefficients are estimated.

The relation of the regression coefficients with the frequency $n\omega_c$ amounts to the phase shift tangent of the n frequency element of the wave train

$$\frac{b_n}{a_n} = \frac{b_n^{(0)} \cos n\omega_c \tau + a_n^{(0)} \sin n\omega_c \tau}{a_n^{(0)} \cos n\omega_c \tau - b_n^{(0)} \sin n\omega_c \tau} = \tan(n\omega_c \tau + \psi_n^{(0)}), \tag{8}$$

where $\psi_n^{(0)} = \arctan\left(\frac{b_n^{(0)}}{a_n^{(0)}}\right) + \eta_n^{(0)}$ is the constant initial phase shift with the frequency $n\omega_c$,

$$\eta_n^{(0)} = \begin{cases} 0, & \text{if } a_n^{(0)} \geq 0 \\ \pi, & \text{if } a_n^{(0)} < 0,\ b_n^{(0)} \geq 0. \\ -\pi, & \text{if } a_n^{(0)} < 0,\ b_n^{(0)} < 0 \end{cases} \tag{9}$$

An equation may be derived from relation (8) to describe the dependence of the time shift $\tau(n\omega_c)$ of the harmonic component with the frequency $n\omega_c$ on the regression coefficients:

$$\tau(n\omega_c) = \frac{1}{n\omega_c}\left[\arctan\left(\frac{b_n}{a_n}\right) + \eta_n\right] - \frac{\psi_n^{(0)}}{n\omega_c}, \tag{10}$$

where

$$\eta_n = \begin{cases} 0, & \text{if } a_n \geq 0 \\ \pi, & \text{if } a_n < 0, \quad b_n \geq 0. \\ -\pi, & \text{if } a_n < 0, \quad b_n < 0 \end{cases} \tag{11}$$

Considering the minimum of functional (7), the estimation of the regression coefficients presupposes that the value of the frequency ω_c is known.

3. ESTIMATION OF THE OSCILLATION WAVE TRAIN REGRESSION MODEL COEFFICIENTS

The linear model for estimating regression coefficients in matrix notation is the following:

$$\mathbf{v} = \mathbf{B}\gamma + \varepsilon, \tag{12}$$

where $\mathbf{v}$ is the column vector of dimension $(N \times 1)$ of the oscillation wave train samples,

$$\mathbf{v} = [u_1, u_2, \ldots, u_N]^T, \tag{13}$$

B is the regression matrix of dimensionality $(N \times (2m+1))$:

$$B = \begin{bmatrix} 1 & \cos\omega_c T_s & \sin\omega_c T_s & \dots & \cos m\omega_c T_s & \sin m\omega_c T_s \\ 1 & \cos\omega_c 2T_s & \sin\omega_c 2T_s & \dots & \cos m\omega_c 2T_s & \sin m\omega_c 2T_s \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & \cos\omega_c NT_s & \sin\omega_c NT_s & \dots & \cos m\omega_c NT_s & \sin m\omega_c NT_s \end{bmatrix}, \quad (14)$$

$\gamma = [c_0, a_1, b_1, \dots, a_n, b_n, \dots, a_m, b_m]^T$ is the column vector of dimension $((2m+1)\times 1)$ of regression coefficients, and ε is the column vector of dimension $(N \times 1)$ of the errors in the signal model and in the quantization of the samples. The estimation of the parameters is confined to minimizing the square form considering the regression coefficient vector.

$$[\mathbf{v} - \mathbf{B}\gamma]^T [\mathbf{v} - \mathbf{B}\gamma] \to \min_{\gamma}. \quad (15)$$

The system of standard equations is the following:

$$\left(\mathbf{B}^T\mathbf{B}\right)\gamma = \mathbf{B}^T\mathbf{v}, \quad (16)$$

whereas the solution of equation system (16) can be written as follows:

$$\hat{\gamma} = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{v}, \quad (17)$$

where $\hat{\gamma} = \left[\hat{c}_0, \hat{a}_1, \hat{b}_1, \dots, \hat{a}_n, \hat{b}_n, \dots, \hat{a}_m, \hat{b}_m\right]^T$ is a column vector of dimension $((2m+1)\times 1)$ of the regression coefficients' estimations.

The time shift $\hat{\tau} \in [0, T_s)$ of the oscillation wave train can be calculated using a relation similar to Eq. (10), only the regression coefficients must be replaced with their estimations. According to experimental research, the amplitudes of the higher harmonies $A_n = \sqrt{a_n^2 + b_n^2}$, $n = 2, 3, \dots, m$ with the frequencies $n\omega_c$ are nearly two hundred times lower than the amplitude $A_1 = \sqrt{a_1^2 + b_1^2}$ of the main harmonic with the frequency ω_c. It is therefore reasonable to determine the time shift only according to the wave train filling frequency (the main oscillation frequency is ω_c). This allows us to substantially decrease the effect of the additive noise on the accuracy of the time shift measurements. The estimate of the time shift is determined by the regression coefficients at the frequency ω_c according to the expression:

$$\hat{\tau} = \frac{1}{\omega_c}\left[\arctan\left(\frac{\hat{b}_1}{\hat{a}_1}\right) + \hat{\eta}_1\right], \quad (18)$$

where

$$\hat{\eta}_1 = \begin{cases} 0, & \text{if } \hat{a}_1 \ge 0 \\ \pi, & \text{if } \hat{a}_1 < 0, \ \hat{b}_1 \ge 0 \\ -\pi, & \text{if } \hat{a}_1 < 0, \ \hat{b}_1 < 0 \end{cases}. \quad (19)$$

The example in Fig. 3 shows the graphic results of matching the regression function (3) with the help of the least-squares method to the samples of two oscillation wave trains with different time shifts relative to the discretization moments.

Numerical techniques were used to study the influence of the frequency inconsistency between the theoretical model (3) and the actual oscillation wave train on the nonlinearity of the interpolative measurement. The dependence of the nonlinearity on the number of N wave train samples used in the estimations was also considered. When the estimation of the regression coefficients with different time instants $\tau \in [0, T_s)$ was modeled on a computer, the wave train filling frequency was $f_c = \frac{\omega_c}{2\pi} = 27.9$ MHz, and the sampling frequency was 100 MHz (the sampling period was $T_s = 10$ ns). The errors in the sample's quantization were not taken into account; i.e., it was asserted that the values of the samples during the discretization were exactly equal to the instant values of the oscillation wave train.

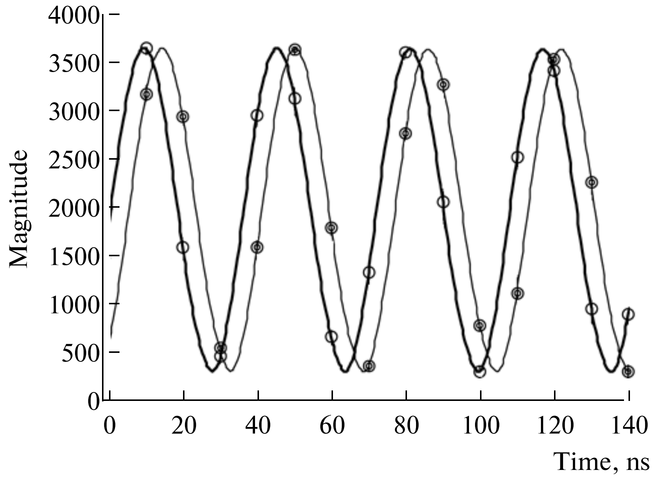

Fig. 3. Results of adjusting the regression function to samples of two wave trains with different time shifts relative to the sampling instants (the sampling frequency is 100 MHz).

According to the results of the computer modeling, the mean degree of the error in estimating the time shift $\Delta\tau = \hat{\tau} - \tau$ does not depend on the actual event's occurrence period if there is no frequency mismatch between the modeled and the actual oscillation wave train:

$$E[\Delta\tau] = E[\hat{\tau} - \tau] = \text{const}, \quad \forall\tau \in [0, T_s). \tag{20}$$

This means that a systematic error dependent of τ of the nonlinearity of the interpolative measurement is absent and therefore we can adopt the estimation (18) of time shift τ as an interpolative estimation of the event occurrence instant.

The curves of the dependence of the error in the interpolative measurement $\Delta\tau = \hat{\tau} - \tau$ upon the event occurrence instant $\tau \in [0, T_s)$ shown in Fig. 4 allow us to conclude that, if there is a mismatch between the values of the frequencies of the regression model and the actual oscillation wave train, a systematic inaccuracy in the interpolative measurement will occur nonlinearily dependent on the event occurrence instant. In the case of a frequency mismatch, the nonlinearity error in the interpolative measurement $\Delta\tau = \hat{\tau} - \tau$ will also depend on the number of N samples used. In the curves in Fig. 4, the nonlinearity error does not exceed ± 1.0 ps. If the frequencies mismatch, the nonlinearity error will not depend on the number of N wave train samples processed.

4. ESTIMATION OF THE BASIC FREQUENCY OF THE OSCILLATION WAVE TRAIN

If the regression model's frequency is inconsistent with the main frequency of the actual wave train, an inaccuracy of the interpolative measurement nonlinearity dependent on the event occurrence instant appears. According to Fig. 4, to minimize the nonlinear inaccuracy, the value of the main wave train frequency ω_c should be known precisely enough. This frequency may be calculated in a variety of ways, for example, using modern digital spectral analyzers in the course of analyzing an oscillation wave train at the output of a low pass Chebyshev filter (see Fig. 1). Let us consider in brief the estimation of the wave train filling frequency ω_c according to samples of a finite sequence of M wave trains.

The number of N samples of a wave train does not suffice to evaluate the filling frequency accurately enough. Nonetheless, to evaluate this frequency, MN samples of a finite sequence of M wave trains generated at the output of the converter (Fig. 1) during the timing of a periodic event flow may be used. A wave train $u(t)$ is generated for each event at the output of the converter and then discreted and digitalized. For more accurate estimation, the period of the event repetition should be aliquant to the sampling period. For a more detailed consideration of selecting an event repetition period, see study [4].

A sequence of identically shaped M wave trains $u(t)$ can be presented as follows:

$$y(t) = \sum_{i=0}^{M-1} u(t - iT_R), \tag{21}$$

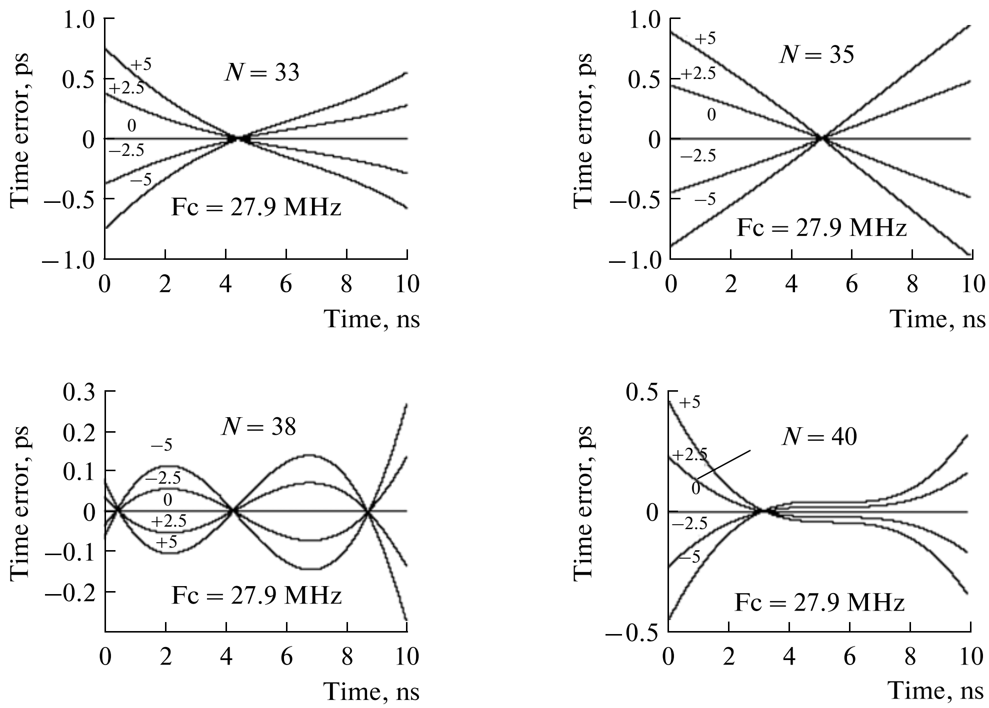

Fig . 4. Dependence of the interpolative measurement error $\Delta\tau = \hat{\tau} - \tau$ upon the event occurrence instant $\tau \in [0, T_s)$, the number of N samples, and the level of the frequency mismatch between the regression model and the oscillation wave train (the numbers above the relevant lines indicate the level of the frequency mismatch in kHz).

where T_R is the period of the event repetition. If the period is not known beforehand, the spectral approach (a sufficiently detailed consideration of which is given in study [4]) can be used to evaluate this period according to all the MN samples.

The complex Fourier spectrum of a single oscillation wave train is determined according to the following equation:

$$U(j\omega) = \int_{-\infty}^{\infty} u(t)e^{-j\omega t}dt, \quad (22)$$

where $j = \sqrt{-1}$ is an imaginary unit. Using MN samples of a sequence of M wave trains (21), the estimation of the complex Fourier spectrum of a single oscillation wave train is expressed as follows:

$$\hat{U}(n\Delta\omega) = T_s \sum_{k=1}^{MN} y(t_k)e^{-jn\Delta\omega\,(t_k \bmod T_R)}, \quad n = \overline{0, Z-1}, \quad (23)$$

where $\{y(t_k)\}_{k=1}^{MN}$ is a sequence of samples of M wave trains during the sampling instants $\{t_k\}_{k=1}^{MN}$, and $\Delta\omega$ is the step of the uniform frequency grid where the estimation of the Fourier spectrum of a single wave train is reconstructed, Z is the number of nodes of the frequency grid. To avoid spectral peaks at null frequency, the mean value of the sequence should be subtracted from the values of a sequence of samples before determining the spectrum estimation (23). The main frequency of the wave train oscillations (filling frequency) can be estimated considering the position of the maximum of the amplitude spectrum on the frequency axis (see Fig. 5):

$$\hat{\omega}_c = \arg\max_{\forall\omega>0} |U(j\omega)|. \quad (24)$$

The following sequence of operations can be underlined for the position determining the position of the peak of discrete amplitude spectrum on the frequency axis:

1) The discrete Fourier transformation (23) is calculated with a very small frequency interval $\Delta\omega$. The mean value of the sample sequence is provisionally subtracted from the sequence itself.

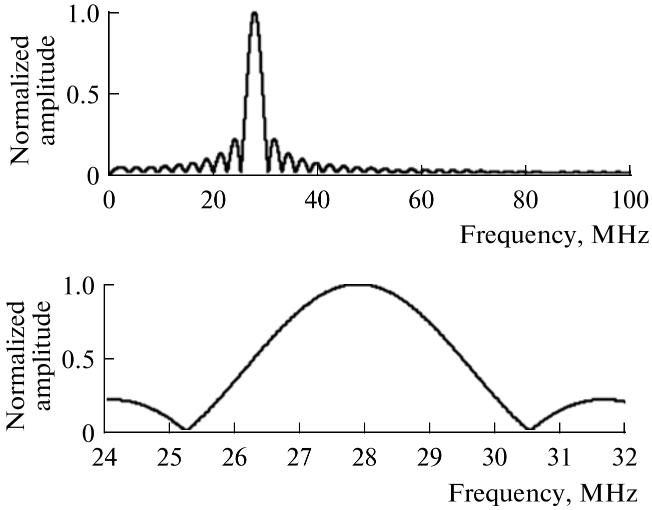

Fig. 5. Reconstructed amplitude spectrum of a single wave train of 25 oscillation periods (upper curve) and its enlarged part in the peak area (lower curve). The wave train filling frequency, i.e., 27.9 MHz, determines the position of the peak amplitude spectrum along the frequency axis.

2) The peak discrete amplitude spectrum (the discrete Fourier transformation module) is searched for along the frequency axis as the number of spectral samples of the highest amplitude:

$$n_{\max} = \arg\max_{n} \left|\hat{U}(n\Delta\omega)\right|. \tag{25}$$

3) The spectral peak position must be aligned by the parabolic interpolation for three spectral samples, and the samples of the highest amplitude should be put in the middle. This alignment is necessary since the peak spectrum may be actually positioned between two fellow spectral samples as the spectrum has a discrete frequency. The necessary frequency correction $\Delta\Omega$ for more precise determination of the spectral peak is calculated according to the following equation:

$$\Delta\Omega = \frac{\left|\hat{U}[n_{\max}-1]\right| - \left|\hat{U}[n_{\max}+1]\right|}{2\left(\left|\hat{U}[n_{\max}-1]\right| - 2\left|\hat{U}[n_{\max}]\right| + \left|\hat{U}[n_{\max}+1]\right|\right)}\Delta\omega. \tag{26}$$

4) The spectral sample number $n_{\max}$ is translated into the frequency, the value of which is added to the frequency correction $\Delta\Omega$:

$$\hat{\omega}_c = n_{\max}\Delta\omega + \Delta\Omega. \tag{27}$$

5. EXPERIMENTAL RESULTS OF MEASURING EVENT FLOW TIME COORDINATES

The accuracy of the studied interpolation technique can be illustrated with experimental results of determining event occurrence instants. A sequence of valued instants was used to calculate the periods of time between two fellow events and to analyze the values' statistical characteristics.

The statistical characteristics of the estimations of the event flow time coordinates were studied using the *method of the analysis of the fluctuations relative to the linear trend* [6]. Unwanted fluctuations relative to the linear trend are referred to in the scientific literature as *jitters*.

To generate an event flow, a high stability pulse generator was used. The sequence of pulses generated was linked to the flow of periodic events. The interval between the events was about 204.794968 μs, while the total number of events was 160. When an event occurred, a wave train of 25 oscillations with the filling fre-

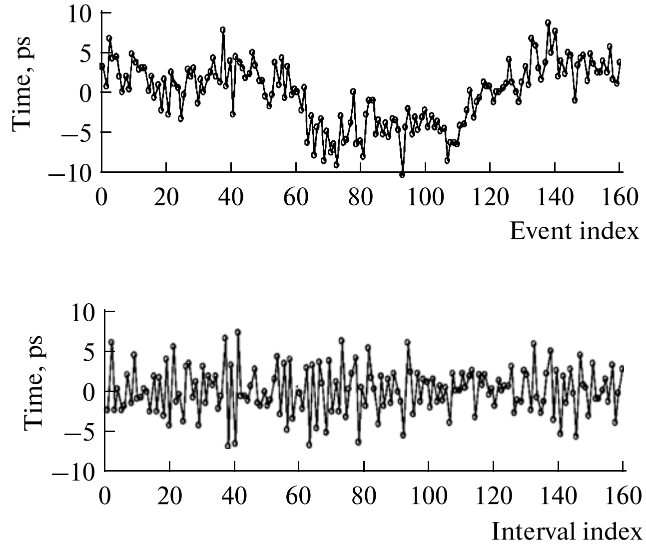

Fig. 6. Results of the actual experiment. Jitter of the event occurrence instants (upper curve) and the sequence of errors in measuring the time intervals between the events (lower curve).

quency $f_c = \frac{\omega_c}{2\pi} = 27.9$ MHz was generated by the converter of the incoming events into wave trains (see Fig. 1). To digitize the instant values of the wave train, a 12-bit AD converter with a sampling frequency of 100 MHz was used. The number of wave train samples used in the processing was 35. However, the samples taken at the initial, i.e., the unsettled, stage of the wave train were not considered, were discarded, and were not used for the processing. The time position of the wave train was estimated using equations (17) and (18). The time interval between the events was estimated as the difference of the time instants when fellow events occurred.

As an example, the fluctuations of the estimations of the event flow time values relative to the linear trend (event occurrence time jitters) are shown in Fig. 6. The estimation of the linear trend of the determined time coordinates of the event flow was displayed in the form of a straight line of regression, the parameters of which were determined using the least-squares method. The lower curve in Fig. 6 shows the sequences of errors in measuring the intervals between the events from the flow. Calculated using the sequence of intervals, the mean square error of the measurement was 2.885 ps.

Figure 7 shows the diagrams of the autocorrelating functions calculated using the data sequences from Fig. 6. It is known that the autocorrelating function of realizing the noncorrelating process has a needlelike shape while the argument value is null. The Dirac delta function, for example, is displayed in the curve in the form of a needle. The autocorrelating function shown in the upper diagram in Fig. 7 is not needlelike, and the jitter of the event occurrence instants is therefore correlated. The jitter *Fractal Dimension* [6] (see the upper curve in Fig. 6) approximates 1.5, which corresponds to the fractal dimension of *random walk* actualizations. An assertion can be made that the event flow produced by the generator and used in the experiment was not strictly periodical but was rather a case of *Palm Flow,* which is also referred to as a recurrent stream of limited residual effect or as a stream with accumulating dispersion of event occurrences.

To analyze the effect of the generator frequency fluctuations on the interval measurement errors, a case of event timing was modeled on a computer. The results of the computer modeled measurement with the source data identical to the actual experiment are given in Figs. 8 and 9, although not a Palm flow but a quasi-periodical stream of events with jittering time values was imitated. The mean square error of the jittering instant was set as 1.0 ps.

The mean square measurement error calculated according to a sequence of intervals amounts to 1.559 ps. Figure 9 shows curves of autocorrelating functions calculated according to the data sequences in Fig. 8. The needlelike shape of the autocorrelating function in the upper curve of Fig. 9 indicates that the event occurrence jitter is not correlated (see the upper diagram in Fig. 8). The jitter's fractal dimension (see the upper diagram in Fig. 8) is 1.995, which corresponds to the fractal dimension of "*white noise*" actualizations.

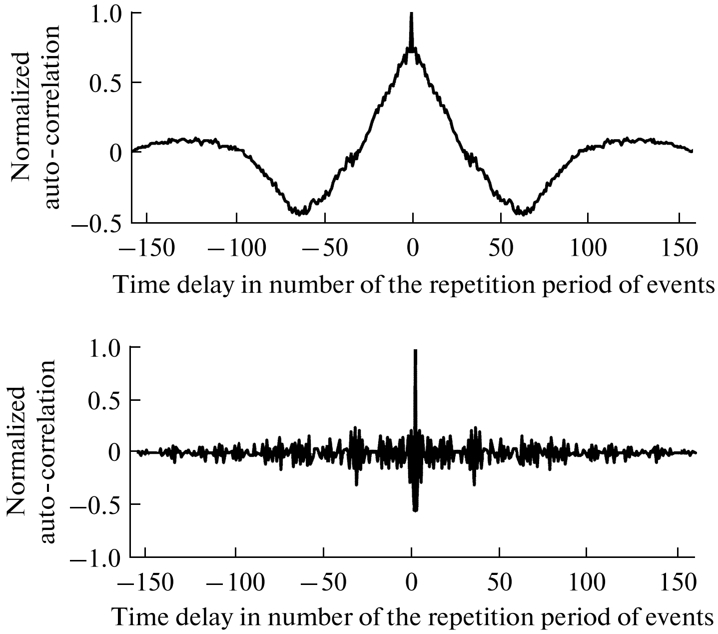

Fig. 7. Results of the actual experiment. The autocorrelation function of the jitter of the event occurrence instants (upper curve) and the autocorrelation function of the sequence of errors in measuring the time intervals between the events (lower curve).

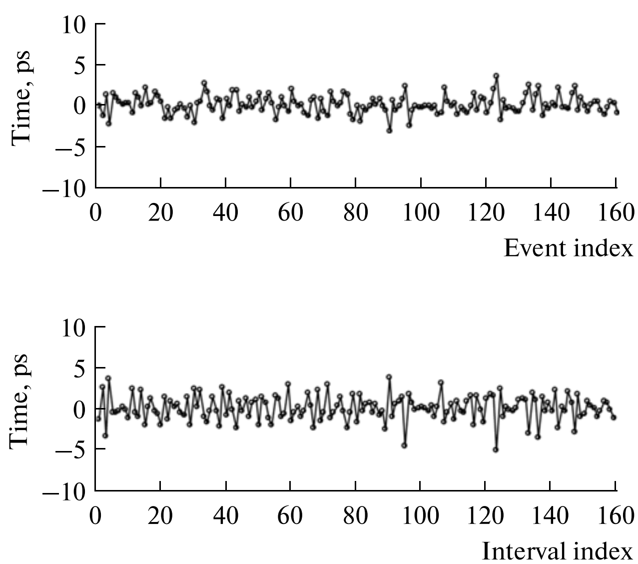

Fig. 8. Computer modeling results. Jitter of the event occurrence instants (upper curve) and the sequence of errors in measuring the time intervals between the events (lower curve).

Comparing the experimental results with the results of the computer modeling, the following conclusion can be reached: the mean square error obtained in the experiment during the measurement of the time intervals by the event timing was 2.885 ps. Nevertheless, this value does not allow one to make an ultimate judgement about the accuracy of the interpolation. The given value is overestimated since it contains not only errors in the interpolation measurement but also errors caused by the unsteady frequency of the generator producing event streams similar to Palm flows. The results of the modeling allows one to assert that, if real measurements of time intervals between events are made, the type of interpolation in question

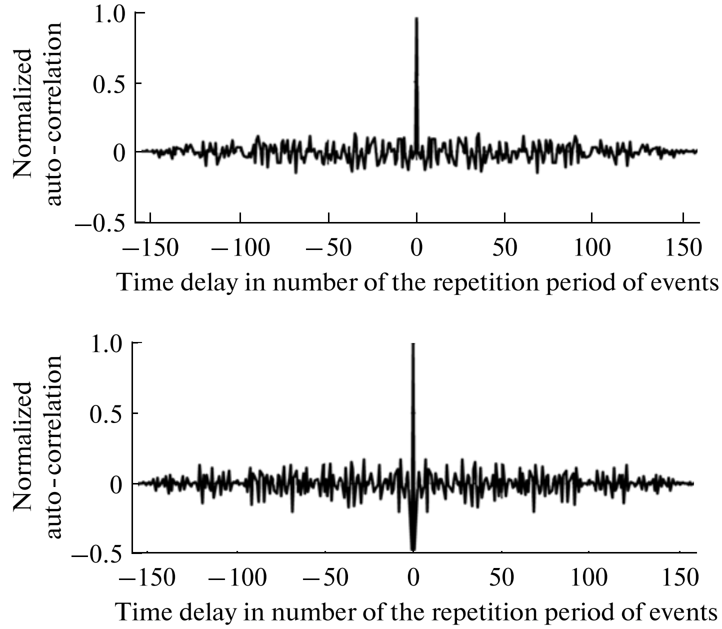

Fig. 9. Computer modeling results. Autocorrelation function of the jitter of the event occurrence instants (upper curve) and the autocorrelation function of the sequence of errors in measuring the time intervals between the events (lower curve).

(using a 12-bit AD converter) will ensure that the mean square error in the separate measurements does not exceed 2.5 picoseconds.

6. CONCLUSIONS

The interpolation measurement of event occurrence instants based on digital processing of oscillation wave train samples is studied. A wave train is generated with each event occurrence. The processing of the samples is the estimation of the coefficients of the regression functions approximating the wave train.

The processing is realized using the regression model of an oscillation wave train with linear information-carrying parameters. The model is a finite sequence of harmonic components with frequencies divisible by the basic oscillation frequency. The time shift of an event occurrence from the beginning of the corresponding discrete interval is determined using the estimated amplitudes of the quaternary harmonic components.

It is shown that the use of linear algorithms of estimating the parameters of the regression model of the oscillation wave train allows one to ensure the picosecond accuracy of the interpolation measurements.

It is shown that the considered type of interpolation does not require extra correction of the obtained estimations of the oscillation wave train time position. The latter estimations can therefore be used directly as interpolation estimations of event occurrence instants, which allows one to avoid the provisional identification of the interpolator's transfer function.

Numerical approaches were used to study the effect of the frequency mismatch between the theoretical and the actual oscillation wave train on the nonlinearity of the interpolation measurement.

The accuracy of the interpolation is proved with the specified experimental results of measuring the time intervals between events of the flow with an inconsistency of about several picoseconds.

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