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Analytic Model and Bilateral Approximation for Clocked Comparator

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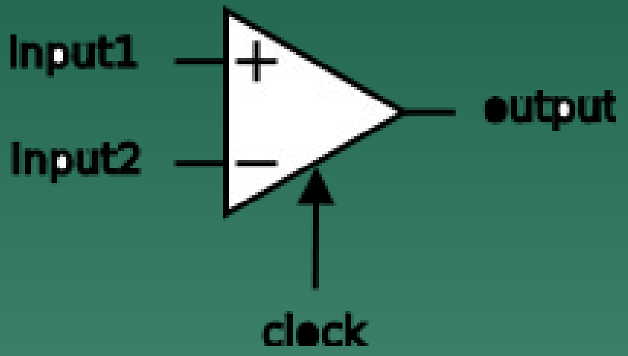
Outline

- Introduction
- Scope of the work
- Clocked comparator model
- Systems differential equation
- Analytical solution of systems diff equation
- Experimental results
- Conclusion

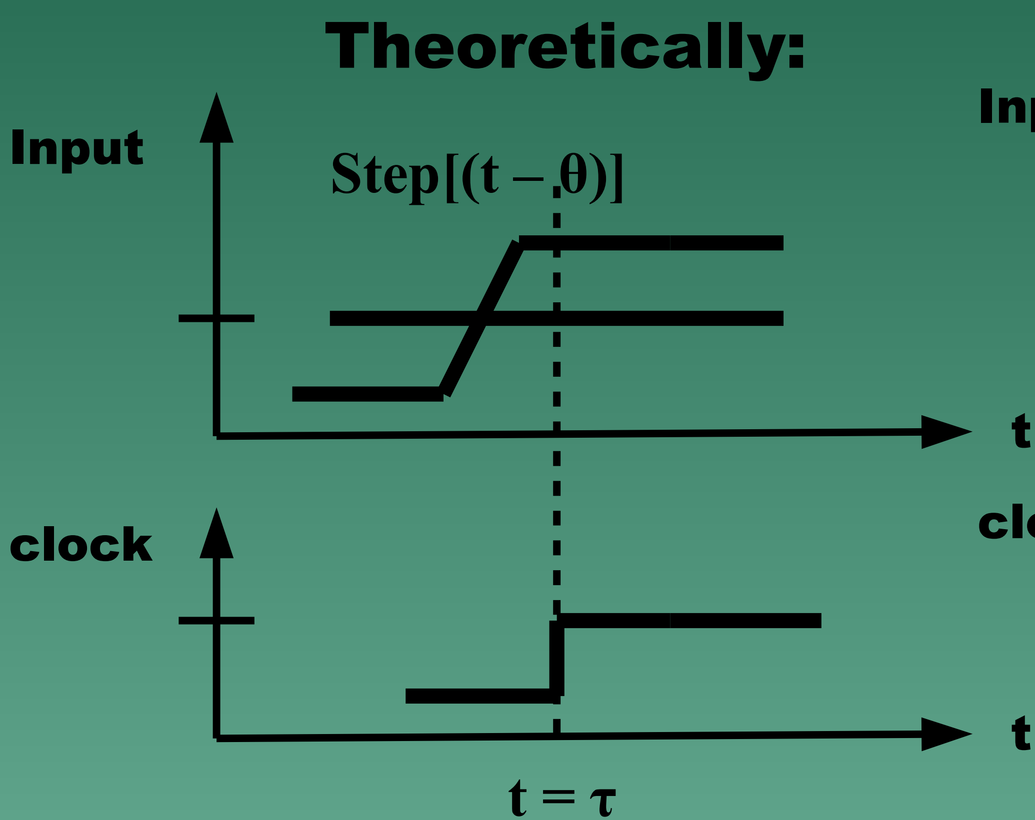
Introduction

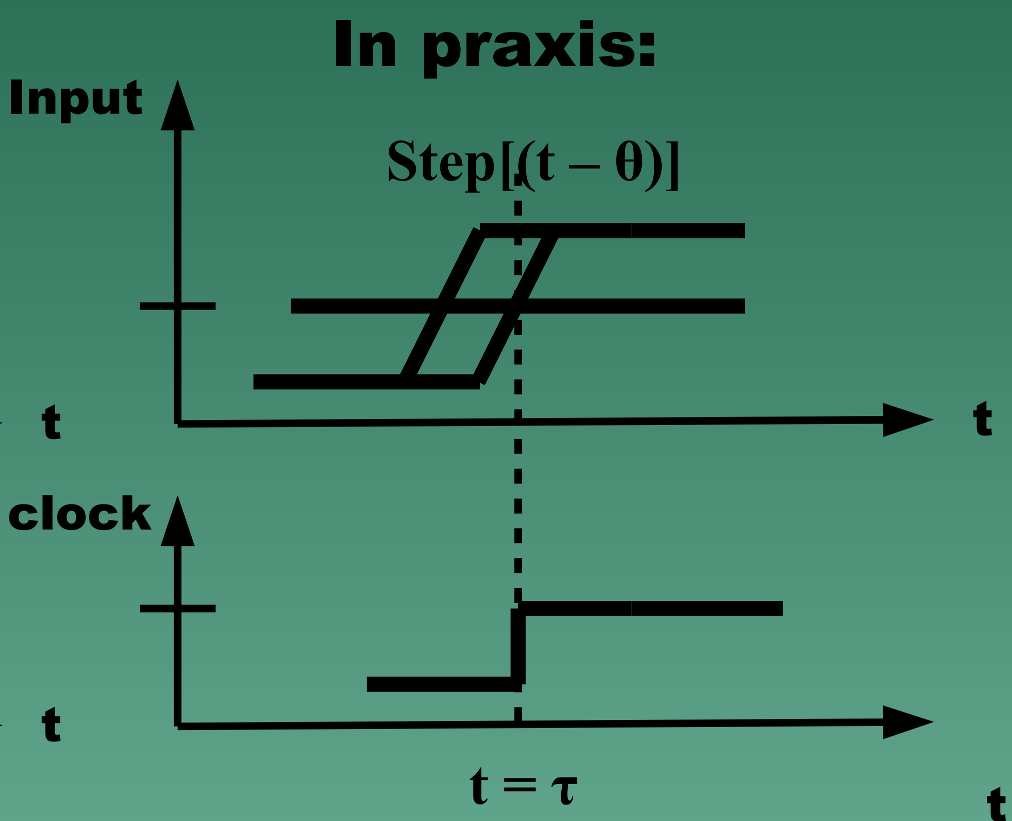
- **Clocked comparators are used in:**
  - **ADC**
  - **Memory devices**
  - **Mixed-signal systems**
  - **...**
- **Comparators' main parameters are:**
  - **Propagation Delay, [µs,ns]**
  - **Offset Voltage, [mV]**
  - **<u>Critical threshold curve</u>**
  - **...**



Motivation

Knowledge about comparators' characteristics allows to build more flexible designs.



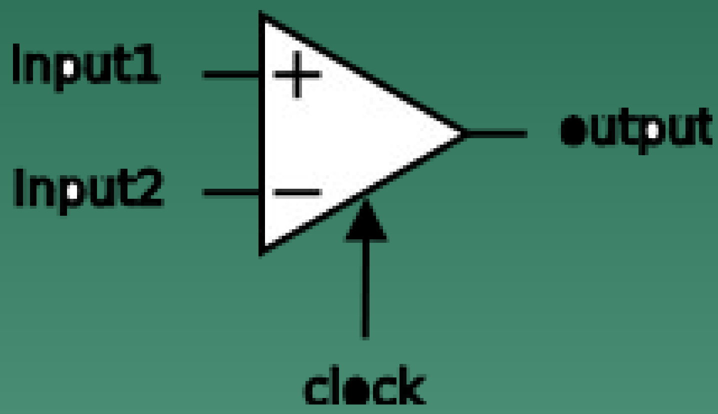


Scope of the work

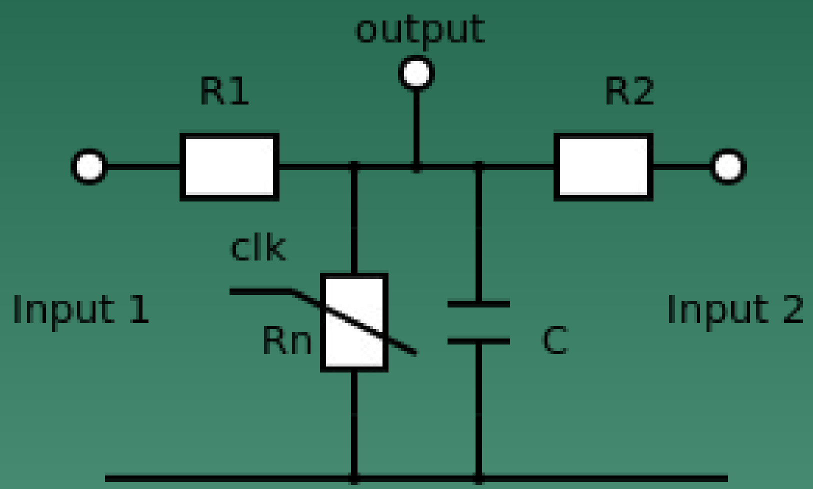
- To offer a model of a clocked comparator
- To describe the simplified model with the first order differential equation and solve it analytically
- To offer bilateral approximation for models described by higher order differential equations
- To get comparator equivalent transition characteristics
- To confirm the results in an experiment

Simplified model with parametric resistance

Clocked comparator



Simplified equivalent circuit

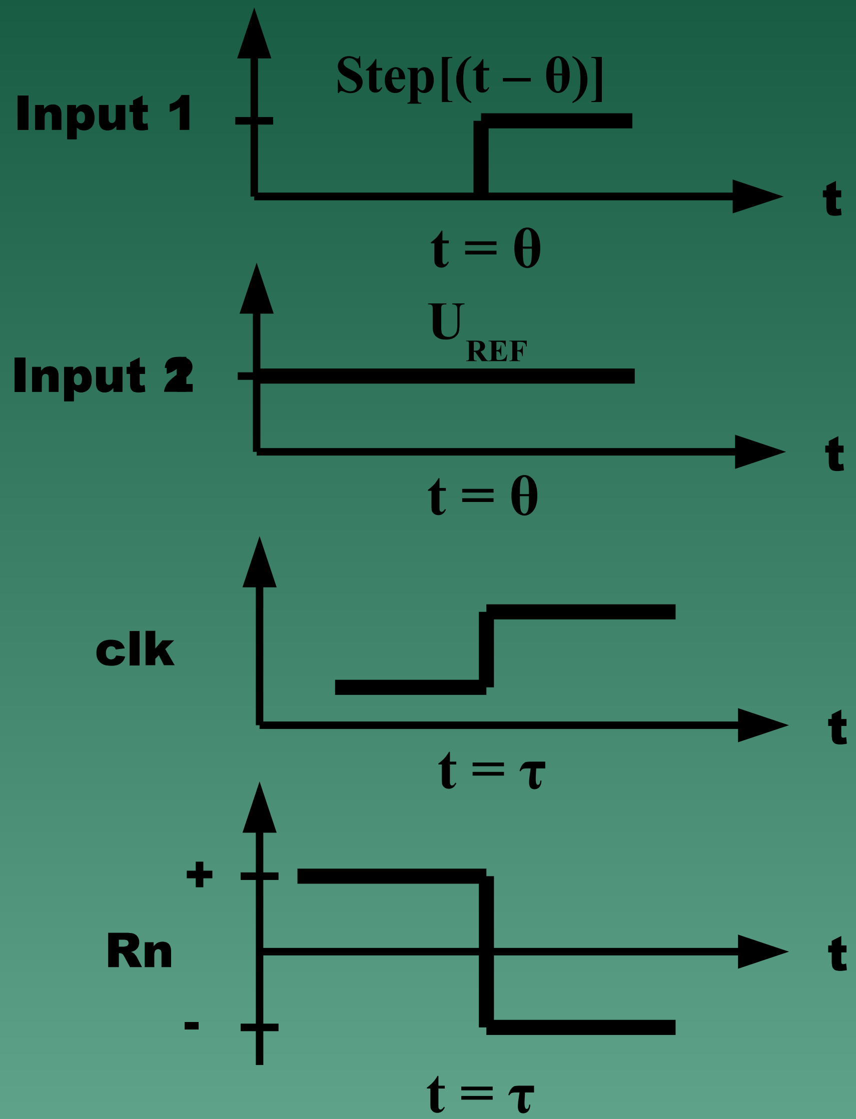


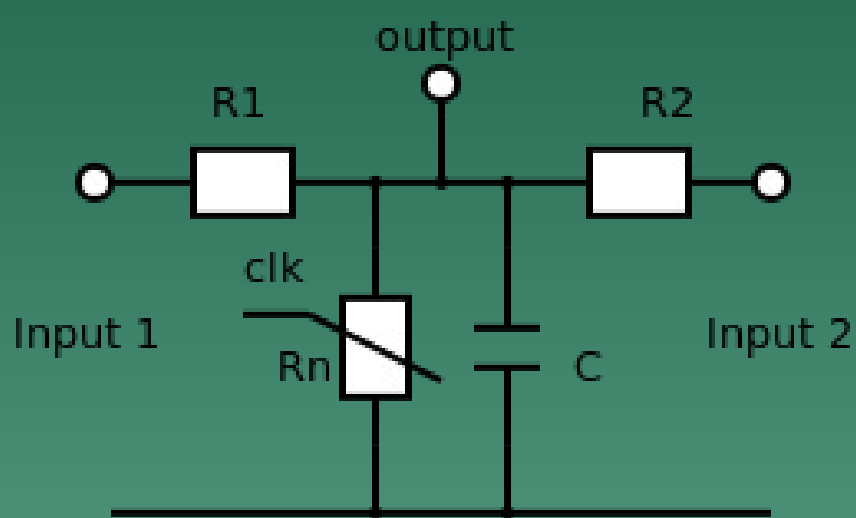
R1, R2 – input resistances

Rn – variable resistance witch changes the sign when the clock is active

C – parasistic capacitance

One step closer to the description of the model

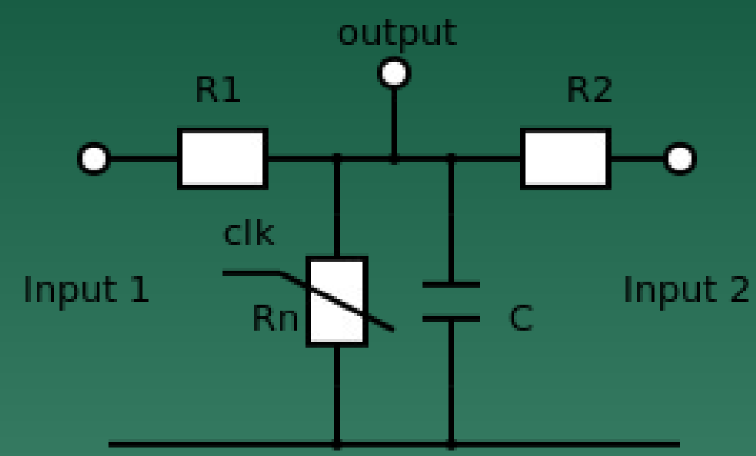




Differential equation of the model

First order ordinary differential equation

$$y'[t]+a_1 y[t]=a_2 U_{ref}+a_3 \mathrm{Step}[(t-\theta)]$$



If R1, R2 $\to\infty$

then the term $\to 0$

$$a_1=\frac{1}{R_n C}+\boxed{\frac{R_1+R_2}{R_1 R_2 C}},\qquad a_1\approx\frac{1}{R_n C}$$

$$a_2=\frac{1}{R_2 C}\qquad a_3=\frac{1}{R_1 C}$$

— **The only parametric coefficient in the equation, that changes its' sign, when the clock is active**

Equation of the model

$$y'[t]+a_1 y[t]=a_2 U_{ref}+a_3 \mathrm{Step}[(t-\theta)]$$

Constants: $a_3=\dfrac{1}{R_1 C}\Rightarrow A$ $\quad a_2=\dfrac{1}{R_2 C}\Rightarrow B$

Variable coefficient: $a_1\approx\dfrac{1}{R_n C}\Rightarrow K[(t-\tau)]$

Equation with time varying coefficient K(t):

$$y'[t]-\mathrm{K}[(t-\tau)]y[t]=A\cdot\mathrm{Step}[(t-\theta)]+B\cdot U_{REF}$$

$K[(t-\tau)]$ – The coefficient changes the sign when $\mathbf{t}=\boldsymbol{\tau}$

Characteristic function

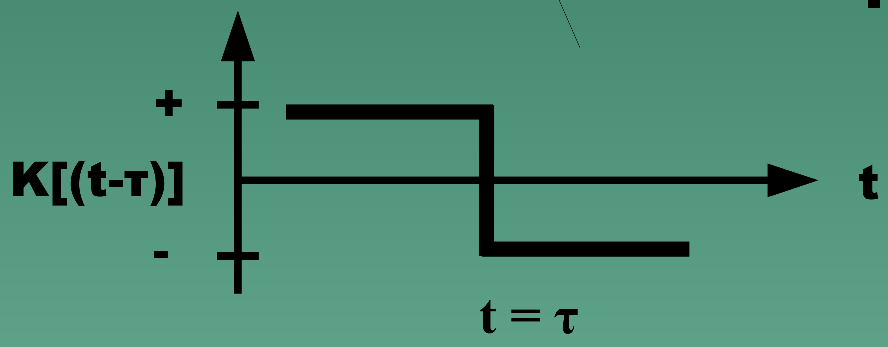
To solve the differential equation we need to choose characteristic function of comparator

$$K[(t-\tau)]$$

$$y'[t]-K[(t-\tau)]\,y[t]=A\cdot Step[(t-\theta)]+B\cdot U_{ref}$$

?

Possibilities:
Linear function: $t_0 - t$
Non-linear: arctan(t), arctanh(t)



The chosen function:

$$K[(t-\tau)]=r\tanh[r(t-\tau)]$$

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Solution to obtain critical threshold

1 Equation is solved considering initial conditions:
$y[t_0] = 0$

2 Limit when $t_0 \to -\infty$ is found

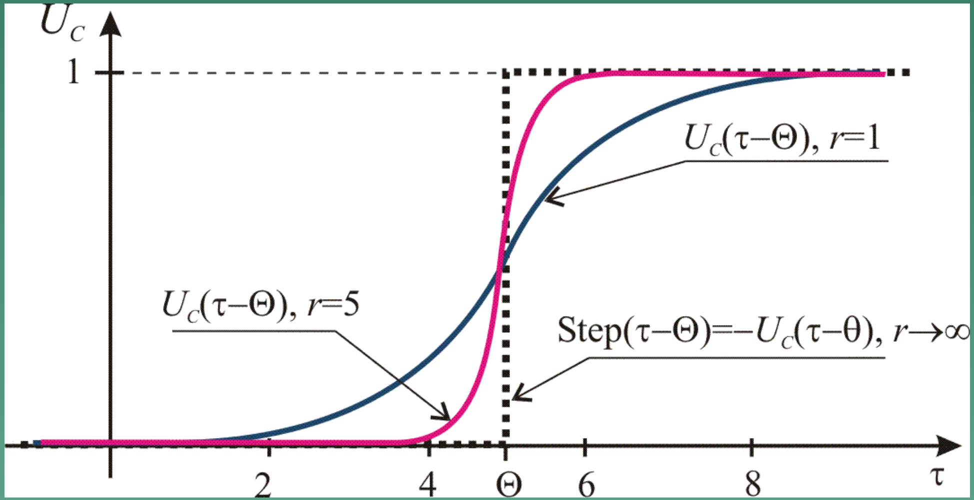
3 Reference level U_{REF} is obtained by solving $y_{-\infty} = 0$

4 Limit when $t \to \infty$ gives the expression of the critical threshold

$$U_C(\tau) = \frac{A\left(\pi - 4\left[\tanh\left[\frac{r(\theta-\tau)}{2}\right]\right]\right)}{2B\pi}$$

$$A = B = 1$$

$$\Theta = 5$$



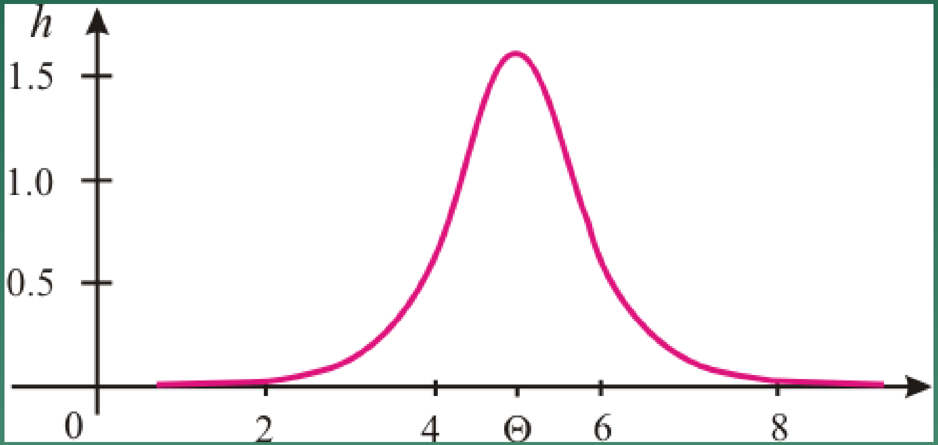
Impulse response

If critical threshold is considered as the transition characteristic, then first derivative illustrates impulse response

$$h(\tau) = \frac{A r \,\mathrm{sech}[r(\theta - \tau)]}{B\pi}$$

$$A = B = 1$$

$$\Theta = 5, r = 5$$



A response to any arbitrary signal could be acquired by using composition of input signal and impulse response function

$$V(\tau) = \int_{-\infty}^{\infty} \mathrm{h}(\tau - \xi)\,\mathrm{f}(\xi)\,d\xi$$

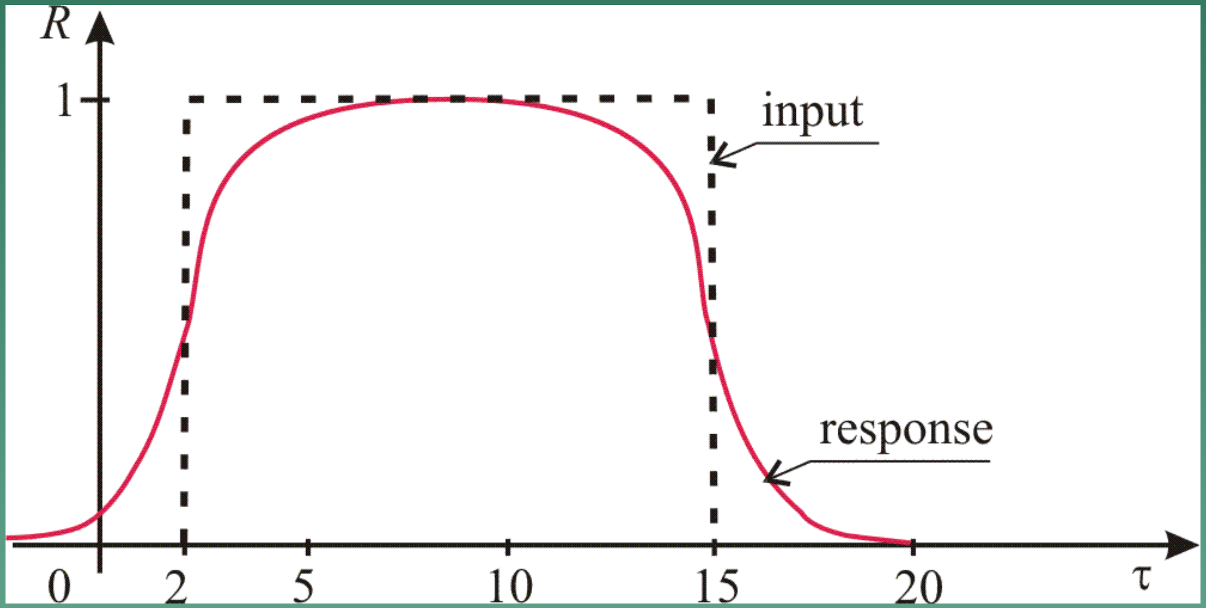
Example: Rectangular pulse

$$R(\tau)=\int_{-\infty}^{+\infty}(\mathrm{Step}[t-2]-\mathrm{Step}[(t-15)])\,\frac{\mathrm{sech}[\tau-t]}{\pi}\,dt=$$

$$=\frac{2}{\pi}\left(\arctan\left[\tanh\left[\frac{15-\tau}{2}\right]\right]-\arctan\left[\tanh\left[\frac{2-\tau}{2}\right]\right]\right)$$

$$A=B=1$$

$$\Theta=5$$



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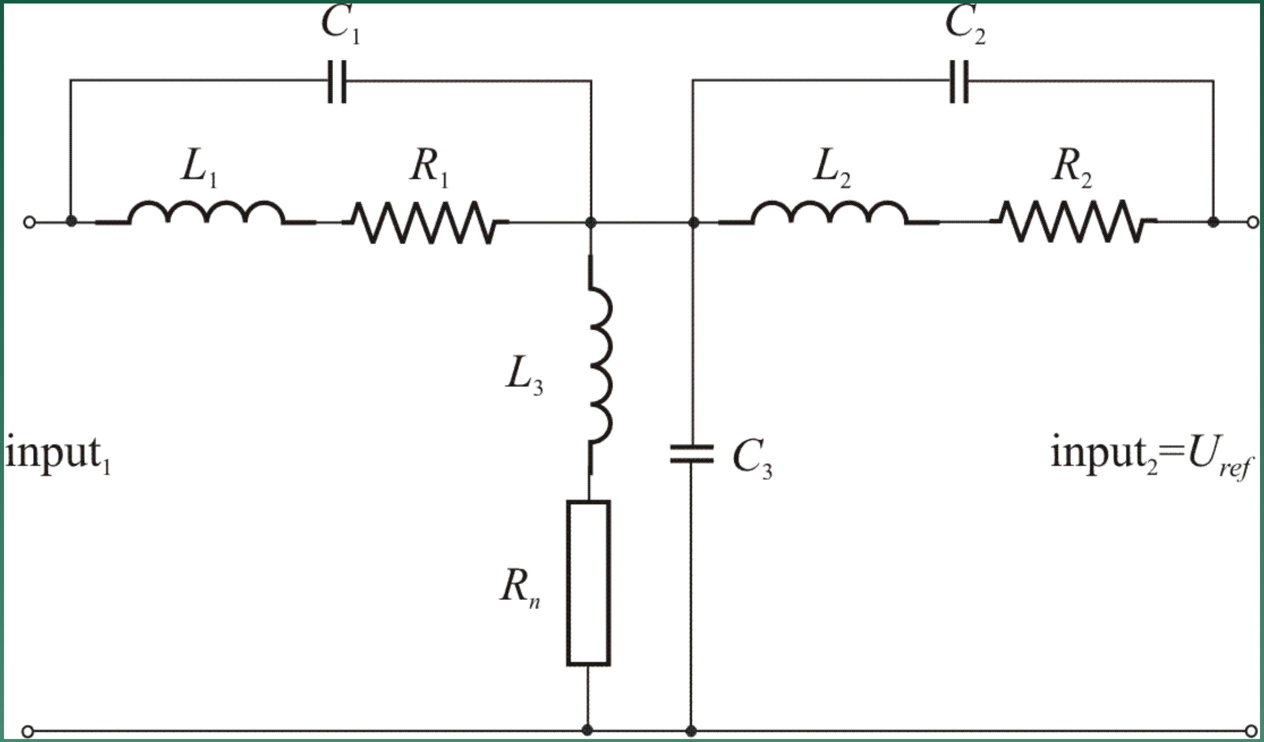
Higher order model



$$a_N y^{(N)}[\tau]+a_{N-1} y^{(N-1)}[\tau]+\ldots+a_0 y[\tau]=f(\tau)$$

***The order depends on the number of the reactive elements in the model**

Bilateral approximation

$$a_N y^{(N)}[\tau] + a_{N-1} y^{(N-1)}[\tau] + \ldots + a_0 y[\tau] = f(\tau)$$

$$a_N z^{(N)}[\tau] + a_{N-1} z^{(N-1)}[\tau] + \ldots + a_0 z[\tau] = 0$$

$$a_N p^{(N)} + a_{N-1} p^{(N-1)} + \ldots + a_0 = 0$$

$$G(\tau-\xi) = \begin{cases} G_+(\tau-\xi) = \sum_{k=0}^{K} A_k \exp(p_k(\tau-\xi)) & if\ \tau \geqslant \xi \\ G_-(\tau-\xi) = \sum_{k=K+1}^{K} A_k \exp(p_k(\tau-\xi)) & if\ \tau < \xi \end{cases}$$

$$U(\tau) = \int_{-\infty}^{\infty} \mathrm{G}(\tau-\xi)\,\mathrm{f}(\xi)\, d\xi$$

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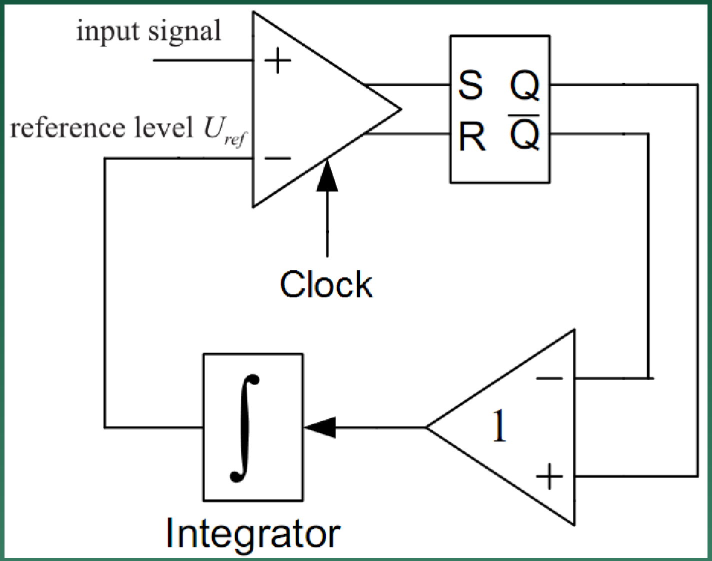
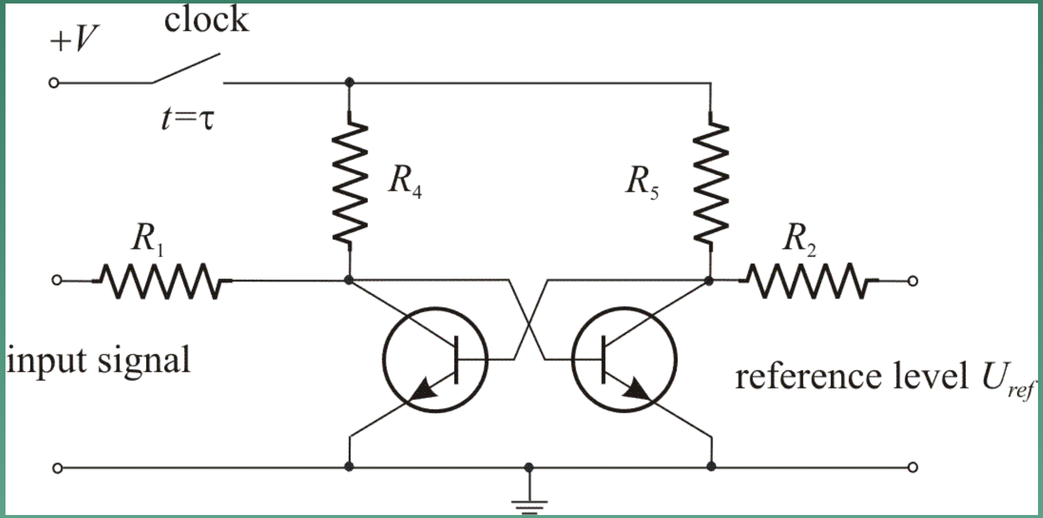
Experimental setup

Comparator's dynamic test setup automatically compensates the reference level and brings the comparator into a metastable state



Comparator's dynamic test setup



Principal electric scheme of the decision stage of the experimental setup

Experimental results

Theoretical and experimental curves

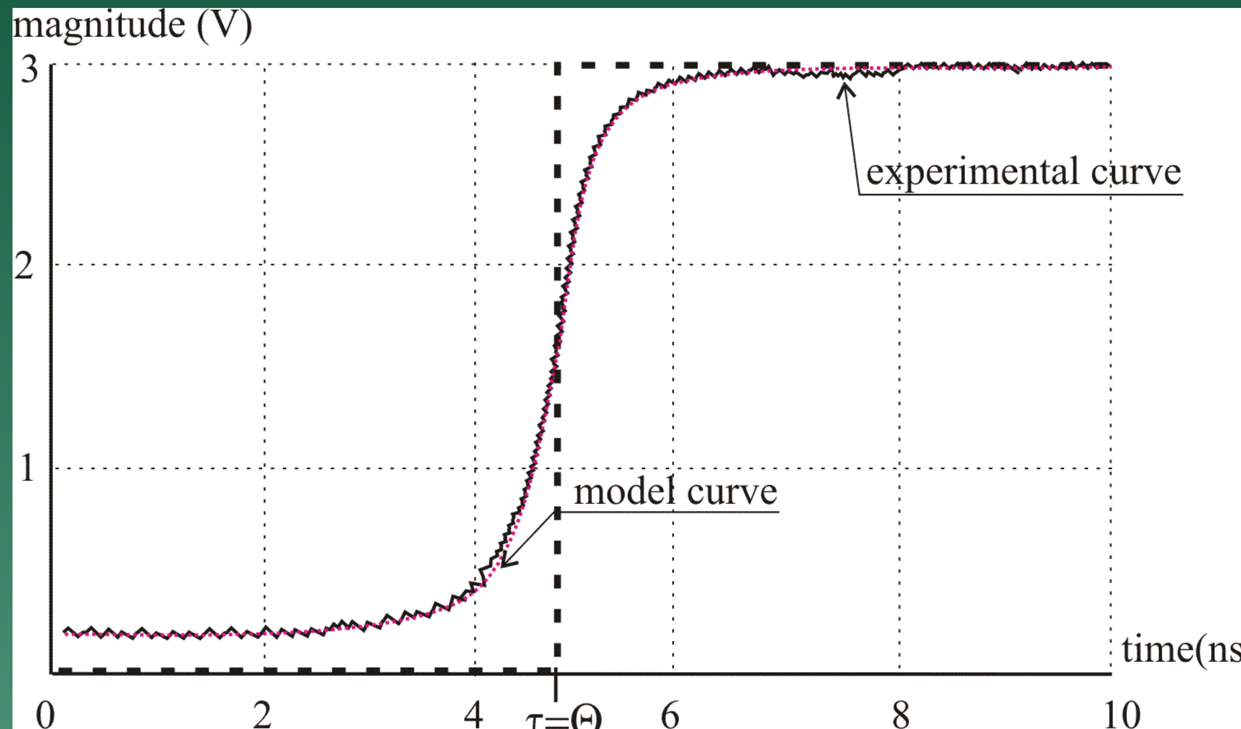
$b = -2.77$

$a = 0.157$

$1/p_1 = -0.34\text{ns}$

$1/p_2 = 0.46\text{ns}$

$\theta = 4.75\text{ns}$



Bilateral approximation:

$$\hat{U}_c(\tau)=\begin{cases} b+a\dfrac{p_1}{p_1-p_2}\exp(p_2(\tau-\theta)) & if\ \tau<\theta \\ \dfrac{p_2}{p_2-p_1}\exp(p_1(\tau-\theta))-1 & if\ \tau<\theta \end{cases}$$

Conclusions

- **A simplified model can be constructed using first-order ordinary differential equation, which can be solved analytically**
- **In cases where higher order models are needed, a method of bilateral approximation can be used to solve corresponding differential equation**
- **Major achievement of the described approach is the possibility to obtain the response of comparator to arbitrary input signal at arbitrary clock point. This response characterize the critical threshold, which provides the metastability of comparator. If this threshold is crossed the logical answer of comparator changes from 0 → 1 or from 1 → 0**

Introduction to equivalent time