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JITTER MEASUREMENT ON THE BASIS OF HIGH-PRECISION EVENT TIMER

Yuri Artyukh, Eugeny Boole

Institut of Electronics and Computer Science, 14 Dzerbenes Str., LV-1006 Riga, Latvia (artyukh@edi.lv)

Abstract

Currently the high-precision event timers represent powerful tools for time measurement in various applications, including jitter measurement. Applied potential of this technology is illustrated by the example of clock jitter measurement and analysis based on application of high-precision event timer. The basic measurement procedures resulting in estimations of commonly used jitter parameters (such as accumulated jitter, period jitter clock-to-clock jitter) are discussed. An approach to informal interpretation of statistical jitter characteristics based on theoretical jitter model and results of computer simulation is offered. Experimental results of jitter measurement and analysis for high-precision clock oscillators confirm assumption that currently the event timing can provide for jitter measurement precision comparable with traditional oscilloscope-based techniques.

Keywords: jitter measurement, event timer, accumulated jitter, statistical jitter analysis.

1. Introduction

As known, there are two basic approaches to jitter measurement and analysis: in frequency domain and in time domains. Jitter measurement in frequency domain is initially related to spectrum analysis, resulting in a continuous phase noise plot over a range of frequencies. Then phase jitter is calculated as the integration of phase noises over a certain spectrum [1]. As for jitter measurement in time domain, basically it is performed using high-performance real-time oscilloscopes with sampling rate up to tens of GHz, and special software that calculates where in time the signal crossed the preset level between the digitized points [2]. Usually in this case the time intervals between two events (so called delta-times) are measured first, and other timing parameters are derived from such measurement. The main advantage of this approach is ability to directly handle high-frequency signals (in GHz range) with high delta-time precision. However as applied to the lower signal frequencies such approach seems too complicated in implementation and too expensive.

Recently, along with this technique, the jitter measurement on the basis of so-called event timers (frequently termed time digitizers) has begun into use [3]. Unlike the real-time oscilloscope, the event timer directly measures the time at instants when some events occur. In jitter measurement the edges of input pulses or signal zero crossings can be considered as an events, providing in digital form full data about their actual position on the time axis. In this respect conceptually the event timers are ideally suited to direct jitter measurement and its further digital analysis. As for performance of the event timers, usually it is specified by precision/speed ratio that varies in a wide range. The highest-speed event timers (such as 9353 100-ps Time Digitizer from ORTEC [4]) offer maximum measurement rate up to 1 GHz and precision about of 85 ps RMS whereas the highest-precision event timers offer picoseconds precision at measurement rate limited down to tens MHz [5].

Applied possibilities of the present-day event timers for jitter measurement and analysis are not sufficiently investigated. In particular, there is indefiniteness in specification of the jitter through event timing, in restrictions for analysing precision in view of limited event timer resolution, etc. We shall discuss part of these problems by example of random jitter measurement and analysis for clock oscillators with use of a high-precision event timer.

2. Technique of jitter measurement by event timer

By definition the event timer supposes specifying the events before their measurement. In view of that a special block (signal conditioner) is used for generating these events when input signal crosses the preset level, i.e. when the signal phase is being incremented by one period. Usually the events are presented in the form of normalised pulses arriving at the timer input. If necessary, additionally this block reduces the rate of event to match it with available measurement rate of event timer. It is assumed that the conditioner is carefully designed and does not introduce significant distortions into original jitter of the signal.

In response to the input events, the event timer generates a series of digital time-stamps $\{t_k\}_0^N$ that reflect actual incrementing of time instants when the signal phase is being incremented by one period (Fig.1).

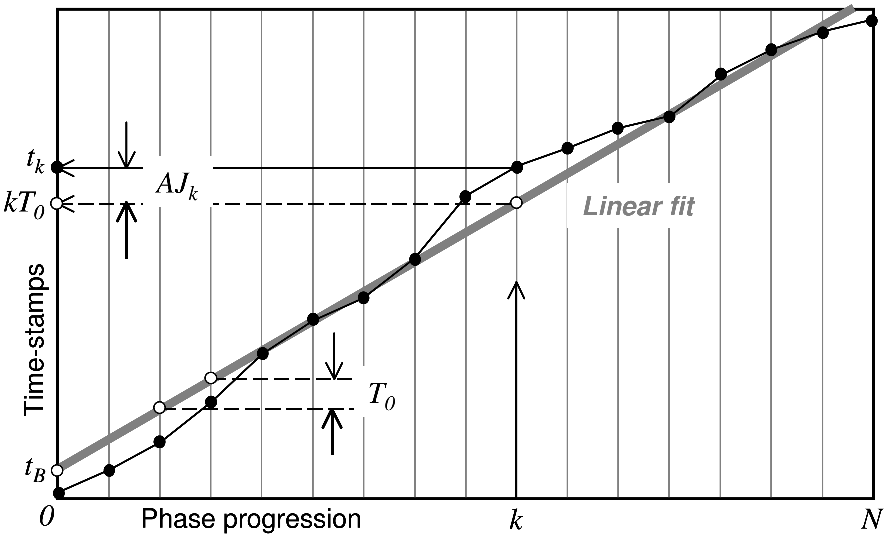

Fig.1. Principle of jitter measurement through event timing

Although the most of event timers offer actually unlimited measurement range, in jitter measurement usually the cycle duration does not exceed one second to be in conformity with general definition of the jitter.

Deviation of the actual incrementing of the time-stamps relatively to their ideally linear incrementing represents dynamics of jitter accumulation. In this case jitter accumulated at every k-th period of input signal can be calculated as residuals of time series data:

$$\{AJ_k = t_k - (kT_0 + t_B)\}^N, \qquad (1)$$

where $(kT_0 + t_B)$ is a linear function that has the best fit to a series of time-stamps, and N – total number of the time-stamps obtained in a single measurement cycle. In this case T_0 is an averaged period of input signal and t_B is a reference time for particular cycle of measurement. Hereafter jitter determined in such a manner, is referred to as A-jitter.

Note that correct choice of the fitting method considerably influences on *A*-jitter determination. To be specific, we perform such fitting on the basis of least-square method that is frequently used in the linear regression approach, and separately for each measurement cycle. In this case the *A*-jitter of a signal (if it is sufficiently stationary) can be evaluated repeatedly with any time-gaps, and without interference from its long-term instability. Additionally such fitting provides for cyclical precise measurements of the period T_0 to determine its deviation vs. real time if required. However it should be taken into account that the fitting, as applied to every measurement cycle, does not allow for possible trends in random jitter accumulation, resulting in understated estimate of the accumulated jitter.

Evaluated in this way *A*-jitter values, arranged in line with period number increasing, represent uniformly sampled *A*-jitter function. Notice that any digital dividing of signal frequency results in reducing of sampling rate, but does not introduce essential distortions in presentation of *A*-jitter function, i.e. it simply will be presented by a less amount of samples.

Using such function, other conventionally used jitter characteristics can be simply derived. Specifically, according to the commonly used metrics, the first differences

$$\{PJ_k=AJ_k-AJ_{k-1}\}^N \qquad (2)$$

conform to so called period jitter (hereafter referred to as *P*-jitter), and the second differences

$$\{CJ_k=PJ_k-PJ_{k-1}\}^N \qquad (3)$$

conform to so called cycle-to-cycle jitter (hereafter referred to as *C*-jitter). In the same manner the differences of higher order can be calculated if desired.

Let us notice that there are a lot of different definitions of the jitter and its particular metrics. For example, by general definition given in ATIS Telecom Glossary, jitter is “the short-term variations of the significant instants of a digital signal from their ideal positions in time” [6]. In this case the term “ideal positions” needs an additional particular definition. So in our case of jitter measurement “ideal positions” of these significant instants are their imaginary positions on a linear fitting of their real positions. Using such approach, it should be taken into account that the fitting, as applied to every measurement cycle, does not allow for possible natural trends in random jitter accumulation. We attribute such trends to more long-term instability of clock sources.

3. Experimental setup

To study actual applied potential of this approach to jitter measurement and analysis, an experimental setup of jitter Analyser has been created.

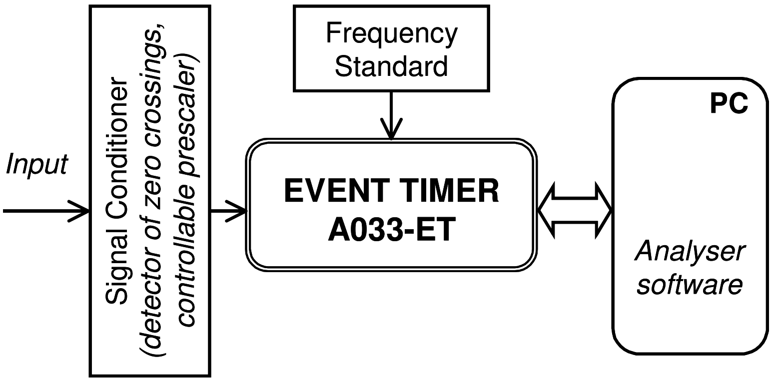

Fig.2. Simplified block diagram of Jitter Analyser

High-precision Event Timer A033-ET [5] represents a core of the Analyser. Typical precision of the A033-ET for time interval measurement is specified in the range of 3.5 to 4 ps RMS, and in our particular case it did not exceed 3.4 ps. Note that such precision is quite comparable with delta-time precision of time measurement for jitter analysis on the base of high-performance real-time DPO700 oscilloscopes from Tektronix, Inc. [7].

To provide the best accuracy, the A033-ET suggests application of external high-performance frequency standards; in our setup the “Thuderbolt GPS disciplined clock” from “Trimble Navigation Limited” was used [8]. As for the measurement rate, the A033-ET supports 20 MSPS burst rate for sequences of up to 2 600 events and 12.5 MSPS burst rate for sequences of up to 16 000 events; the maximum average rate of continuous (gapless) event measurement over a long period of time is 30 KSPS.

The A033-ET is a virtual instrument, allowing application of software tailored to the specific needs. In our case it was experimental software for jitter analysis running under MS Windows. According to the discussed technique of jitter measurement, in single-shot mode of operation the Analyser calculates and shows three basic jitter functions (Fig.3).

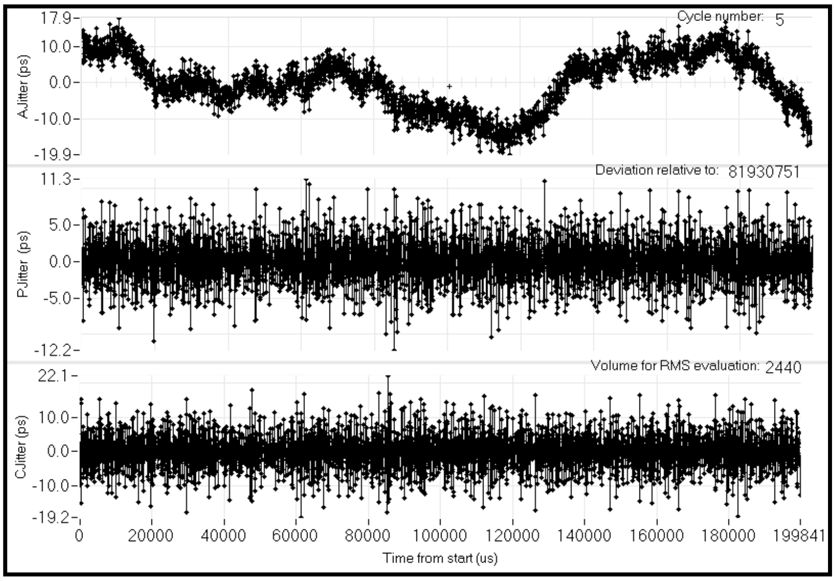

Fig. 3. Crystal clock generator jitter vs. time (1000 clock pulses).
From top to bottom: *A*-jitter, *P*-jitter and *C*-jitter

Generally such jitter plots offer many useful data for qualitative jitter analysis, especially the plot of *A*-jitter. In particular, Figure 3 visually demonstrates its noticeable deviation vs. time in comparison with other kinds of jitter. Similarly to this case, deterministic jitter in the form of repeatable periodic modulations of the jitter functions can be visually detected.

Along with single-shot mode, the Analyser supports mode of repetitive measurement. In this mode the Analyser calculates and shows generalised characteristics of measured jitter from cycle-to cycle, indicating them as functions of time at which every measurement cycle is performed (Fig. 4).

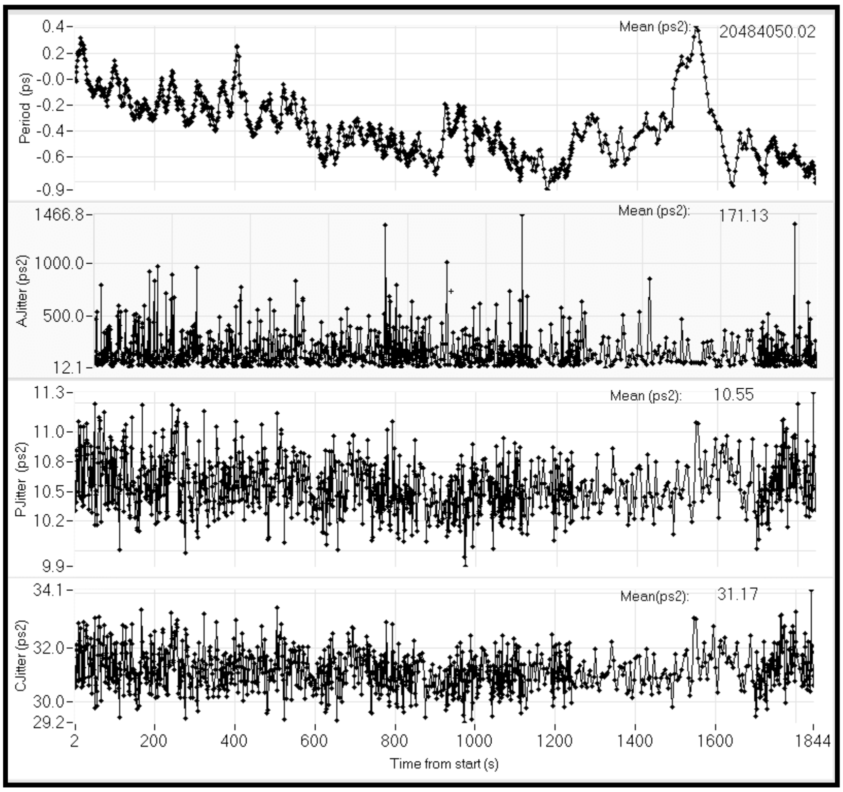

Fig.4. Generalised characteristics of crystal clock generator jitter from cycle-to cycle. From top to bottom: signal period deviation, squared RMS deviations of *A*-jitter, *P*-jitter and *C*-jitter. Observation time: 30 min.

This Analyser's mode permits to detect spurious bursts in jitter measurements and to exclude theirs from further analysis if desired. Additionally the Analyser provides for averaging of these values for a user-defined amount *M* of measurement cycles. As Figure 4 suggests, it is especially necessarily for the *A*-jitter where possible dispersion of S_A estimates from cycle-to-cycle can be very large.

4. Statistical clock jitter analysis

4.1. Theoretical model of jittered oscillator

To better understand physical essence of the mentioned above statistical characteristics, let us consider often used simplified theoretical model of jitter accumulation [9]. In particular, let us suppose that oscillator's clock jitter contains only random components so that the measured in every measurement cycle *k*-th time-stamp (*k*=0, 1, … *N*) can be presented as follows:

$$t_k = kT_0 + \sum_{i=1}^{k} \Delta_i^A + \Delta_k^S , \qquad (4)$$

where Δ_i^A - accumulative jitter component and Δ_k^S - superimposed (non-accumulative) jitter component. Usually the last is caused by temporal instability of output buffer electronics of clock oscillators, and, additionally, in real jitter measurement can include its errors.

Under these assumptions and taking into account equations (1-3), the jitter values AJ_k, PJ_k and CJ_k for every *k*-th time-stamp can be expressed as follows:

$$AJ_k = \sum_{i=1}^{k} \Delta_i^A + \Delta_k^S ; \tag{5}$$

$$PJ_k = \Delta_k^A + (\Delta_k^S - \Delta_{k-1}^S); \tag{6}$$

$$CJ_k = (\Delta_k^A - \Delta_{k-1}^A) + (\Delta_k^S - 2\Delta_{k-1}^S + \Delta_{k-2}^S). \tag{7}$$

Let us assume that the Δ_i^A and Δ_k^S are uncorrelated random values characterised by variances Var(A) and Var(S) respectively. In this case the variances of the above jitter values will conform to:

$$\mathrm{Var}(AJ_k) = k\mathrm{Var}(A) + \mathrm{Var}(S); \tag{8}$$

$$\mathrm{Var}(PJ_k) = \mathrm{Var}(A) + 2\mathrm{Var}(S); \tag{9}$$

$$\mathrm{Var}(CJ_k) = 2\mathrm{Var}(A) + 6\mathrm{Var}(S). \tag{10}$$

As can be seen from equations (8-10), relative influence of the jitter components in total jitter of each jitter kind significantly differs. In particular, the accumulative component much more pronouncedly is presented in A-jitter but the superimposed component – in C-jitter. Generally the relative influence of the superimposed component on total jitter rises according as the order of A-jitter differences increases.

4.2. Experimental evaluation of the model parameters

Let us consider possibilities to evaluate the variances Var(S) and Var(A) of the above model on the basis of experimental data with the aim to predict the jitter characteristics for particular clock oscillator under various conditions of its application.

As can be seen from equations (9, 10), the variances Var(PJ_k) and Var(CJ_k) are independent from the serial number k of signal period. Correspondingly the values of S_P^2 and S_C^2 can be considered as experimental estimates of these variances, i.e. $S_P^2 \cong \mathrm{Var}(PJ_k)$ and $S_C^2 \cong \mathrm{Var}(CJ_k)$ if N is sufficiently large integer. Taking that into account, the Var(A) and Var(S) values can be derived from equations (9,10) as follows:

$$\mathrm{Var}(A) \cong 3S_P^2 - S_C^2; \tag{11}$$

$$\mathrm{Var}(S) \cong 0.5(S_C^2 - 2\,S_P^2). \tag{12}$$

Evidently, the above design equations are applicable if ratio $R = S_P^2/S_C^2$ is in the range from 1/3 (when Var(A)=0) to 1/2 (when Var(S)=0). Otherwise one can conclude that either statistical error of the estimation is too large or the considered theoretical model is not applicable for particular oscillator under test. Specifically, when the ratio R is near to 1/3 or to 1/2, a good statistics is needed to separate one jitter component from another in conformity with equations (11,12).

At first glance one might expect that experimental estimate S_A^2 is more suitable for variance Var(A) evaluation as the accumulative jitter component can be represented in this estimate more apparently. However, it should be taken into account that the A-jitter values are strongly correlated over their sequence and single estimate S_A^2 reflects only some unique realisation of jitter accumulation. Correspondingly only averaged value $\overline{S}_A^2$ of estimates S_A^2 obtained in a lot of measurement cycles may be statistically reliable. This feature makes the estimate S_A^2 not too attractive for variance Var(A) evaluation in practical applications. Additionally, in view of particular defining the A-jitter function with use of time-stamps fitting, there are certain problems to express ratio between $\overline{S}_A^2$ and Var(A) analytically in easy-to-use form.

The above estimate (11) of Var(A) value is attributed to the measured signal period T_0 which can differ for different oscillators, containing different number of original signal periods. To simplify comparison of different oscillators (in terms of their accumulative jitter component), let us normalize Var(A) value by the measured signal period T_0 so that $\mathrm{RMS_N}(A)= \mathrm{Var}(A)/T_0$. In this case jitter variance accumulated during variable period T_M, is equal to $T_M[\mathrm{RMS_N}(A)]$. Thus the experimentally evaluated parameter $\mathrm{RMS_N}(A)$ can be applied for prediction of RMS jitter accumulated during any user-defined time interval T_M.

4.3. Some experimental data and their interpretation

To investigate applied possibilities of the considered approach to jitter analysis and to verify validity of the considered above jitter model, we took for testing two types of high-performance crystal oscillators: clock oscillator (CXO) and voltage-controlled oscillator (VCXO) from the Fordahl-FOQ Group [10]. Each of them represents a hybrid circuit packaged into hermetic holder that contains the oscillator and output buffer electronics. However these devices slightly differ in implementation, and therefore the manufacturer specifies better stability of the VCXO as than that for the CXO.

We have performed the jitter measurement for each type of the oscillators in time-ranges 100 and 200 ms. Periods of the measured signals were 14.084 µs (CXO) and 20.492 µs (VCXO) after preliminary dividing of their original frequencies. Table 1 (2-4 columns) illustrates the squared deviations S_A^2, S_P^2 and S_C^2 that were obtained by averaging measurement data over 1000 cycles. Columns 5-6 of the table represent RMS deviations of the model parameters that are evaluated in conformity with equations (11,12); column 7 represents normalized estimate of the accumulated jitter.

Table 1. Experimental data concerning clock jitter evaluation

1	2	3	4	5	6	7
Oscillator type	S_A^2 (ps^2)	S_P^2 (ps^2)	S_C^2 (ps^2)	RMS(A) (ps)	RMS(S) (ps)	$\mathrm{RMS_N}(A)$ (ps)
VCXO						
T_M= *100 ms*	43.13	10.71	31.65	0.49	2.26	1.17 E-8
T_M= *200 ms*	171.13	10.55	31.17	0.49	2.24	1.17 E-8
CXO						
T_M= *100 ms*	118.66	10.80	32.08	0.57	2.29	2.31 E-8
T_M= *200 ms*	307.74	10.94	32.26	0.75	2.28	3.99 E-8

On the basis of these data one can see:

- Inequality $2S_A^2 >> S_P^2$ clearly indicates that both oscillators under test are distinguished by presence of noticeable accumulative component of total jitter. In a certain sense this criteria seems like the Durbin-Watson statistic.
- The values RMS(A) evaluated for the VCXO actually are the same for two time-ranges in conformity with the theoretical model. However, that does not concern the CXO where process of jitter accumulation seems as considerably non-linear.
- As the data in column 7 indicate, the VCXO is much better than the CXO in terms of jitter accumulation. Especially that concerns great time-ranges. For example, the calculated jitter for 1 sec time-range is 108 and 200 ps (RMS) for the VCXO and CXO respectively.

- In estimations of RMS(*S*) (column 6) the event timer error evidently dominates. Unfortunately this error cannot be precisely separated from oscillator jitter. Nevertheless, comparison of these estimates for the CXO and VCXO indicates that CXO superimposed jitter is greater (by 0.4 ps RMS approx) than that for the VCXO.
- In general, one may conclude that the VCXO offers stability markedly better than that for the CXO in all respects.

5. Summary

Relatively simple unified technique of jitter measurement based on high-precision event timer application is proposed. This technique offers jitter characterization in different views, starting from the directly measured accumulated jitter that is of especial interest for various applications. A new approach to this jitter characterization based on excluding its trends (de-trending) is discussed. This approach allows multiple measurements of the accumulated jitter and its statistical description independently from long-term signal instability.

An approach to statistical jitter analysis based on the simplified theoretical model of jittered clock oscillators is discussed. Such approach provides for correct quantitative interpretation of the measurement results and experimental evaluation of the model parameters. Experimental research confirms ability to evaluate these parameters with femtosecond precision using the event timer with precision about of two picoseconds. However the model not always fully conforms to the real cases, indicating more complicated nature of jitter accumulation for typical clock oscillators.

Generally it seems that the considered technique of jitter measurement could complement traditional oscilloscope-based technique, offering comparable precision, simpler and cheaper solution as applied to the jitter of input signals limited in upper frequency down to hundreds of MHz. Expected advantages of this technique also concern a versatile characterization of clock oscillators in a wide time-range.

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